CAMBRIDGE TECHNOLOGY IN MATHS

Year 12
Exponential and logarithmic functions for the TI-89

CONTENTS

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Example: Solving logarithmic equations

Solve each of the following equations for x:

a $\log_2 x = 5$ **b** $\log_2 (2x - 1) = 4$ **c** $\log_e (3x + 1) = 0$

Solution

a $\log_2 x = 5$

$\therefore\ x = 2^5$

$\therefore\ x = 32$

b $\log_2 (2x - 1) = 4$

$\therefore\ 2x - 1 = 2^4$

$\therefore\ 2x = 17$

$\therefore\ x = \dfrac{17}{2}$

c $\log_e (3x + 1) = 0$

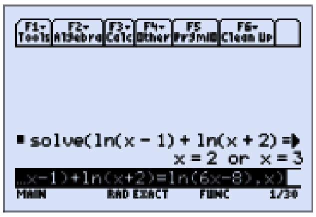
$\therefore\ 3x + 1 = e^0$

$\therefore\ 3x = 1 - 1$

$\therefore\ x = 0$

Using a CAS calculator

Enter **solve**$(\ln(x - 1) + \ln(x + 2) = \ln(6x - 8), x)$



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Example: Solving exponential equations

Given that $y = Ae^{bt}$ and $y = 6$ when $t = 1$ and $y = 8$ when $t = 2$, find the values of b and A.

Solution

When $t = 1, y = 6$

Thus $6 = Ae^{b}$ (1)

When $t = 2, y = 8$

Thus $8 = Ae^{2b}$ (2)

Divide (2) by (1):

$$\frac{4}{3} = e^{b}$$

$$\therefore b = \log_e \frac{4}{3}$$

Substitute in (1):

$$6 = Ae^{\log_e \frac{4}{3}}$$

$$\therefore 6 = \frac{4}{3}A$$

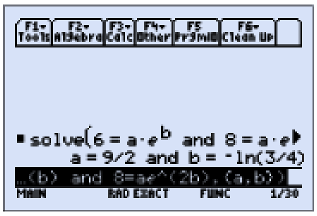
$$\therefore A = \frac{18}{4} = \frac{9}{2}$$

Hence $y = \frac{9}{2}e^{(\log_e \frac{4}{3})t}$ $\left(= \frac{9}{2}\left(\frac{4}{3}\right)^{t}\right)$

$$y \approx \frac{9}{2}e^{0.288t}$$

Using a CAS calculator

Enter **solve**$(6 = ae^{b}$ and $8 = ae^{2b}, \{a,b\})$



Example: Inverses

Find the inverse of the function $f: (1, \infty) \to R$, $f(x) = 2\log_e(x-1) + 3$. State the domain and range of the inverse.

Solution

Consider $x = 2\log_e(y-1) + 3$

Therefore $\frac{x-3}{2} = \log_e(y-1)$

and $y - 1 = e^{\frac{x-3}{2}}$

Therefore $y = e^{\frac{x-3}{2}} + 1$

Hence $f^{-1}(x) = e^{\frac{x-3}{2}} + 1$

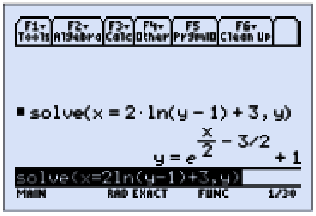
The domain of f^{-1} = the range of f

$= R$

The range of $f^{-1} = (1, \infty)$.

Using a CAS calculator

Enter **solve**$(x = 2\ln(y-1) + 3, y)$



Original location: Chapter 5 Example 26 (p.183-184)

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