# CAMBRIDGE TECHNOLOGY IN MATHS

## *Year 11*
## Functions, relations and transformations for the TI-89

### CONTENTS



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## Example: Solving functions

Consider the function defined by $f(x) = 2x - 4$ for all $x \in R$.

**a** Find the value of $f(2)$, $f(-1)$ and $f(t)$.

**b** For what values of $t$ is $f(t) = t$?

**c** For what values of $x$ is $f(x) \geq x$?

### Solution

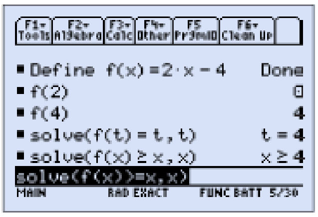
**a**
$$\begin{aligned} f(2) &= 2(2) - 4 \\ &= 0 \\ f(-1) &= 2(-1) - 4 \\ &= -6 \\ f(t) &= 2t - 4 \end{aligned}$$

**b**
$$\begin{aligned} f(t) &= t \\ \therefore 2t - 4 &= t \\ \therefore t - 4 &= 0 \\ \therefore t &= 4 \end{aligned}$$

**c**
$$\begin{aligned} f(x) &\geq x \\ \therefore 2x - 4 &\geq x \\ \therefore x - 4 &\geq 0 \\ \therefore x &\geq 4 \end{aligned}$$

## Using a CAS calculator

Use **Define** with the function $f(x) = 2x - 4$ to find $f(2)$ and $f(t)$, and to solve the equation $f(t) = t$ and the inequality $f(x) \geq x$.



## Example: Restoration of a function

Sketch the graph of each of the following functions and state its range.

**a** $f: [-1, 2] \to R, f(x) = x$

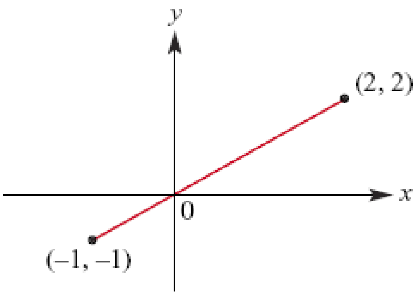
**b** $f: [-1, 1] \to R, f(x) = x^2 + x$

**c** $f: (0, 2] \to R, f(x) = \frac{1}{x}$

**d** $f: R \to R, f(x) = x^2 - 2x + 8$

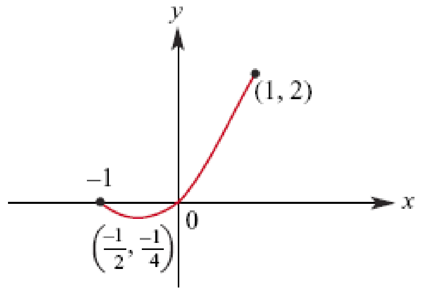
**Solution**

**a**



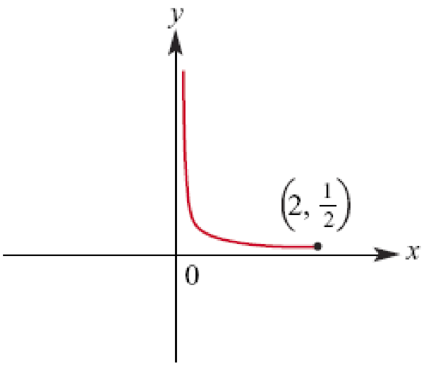

Range is $[-1, 2]$

**b**



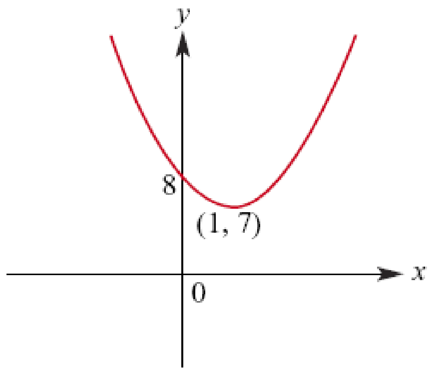

Range is $\left[\frac{-1}{4}, 2\right]$

**c**




Range is $\left[\frac{1}{2}, \infty\right)$

**d**



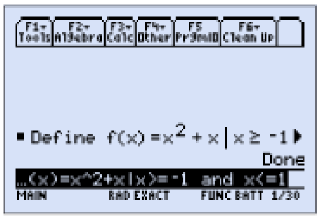

$f(x) = x^2 - 2x + 8 = (x - 1)^2 + 7$

Range is $[7, \infty)$

# Using a CAS calculator

Define the function $f: [-1, 1] \to R,\ f(x) = x^2 + x$.

The graph of $y = f(x)$ is plotted by entering $Y1 = f(x)$ in the Y= screen.



## Example: Inverse functions

Find the inverse function $f^{-1}$ of the function $f(x) = 2x - 3$ and sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ on the one set of axes.

### Solution

The graph of $f$ has equation $y = 2x - 3$ and the graph of $f^{-1}$ has equation $x = 2y - 3$, i.e. $x$ and $y$ are interchanged.

Solve for $y$:

$$x + 3 = 2y$$

and $$y = \frac{1}{2}(x + 3)$$

$$\therefore \quad f^{-1}(x) = \frac{1}{2}(x + 3)$$

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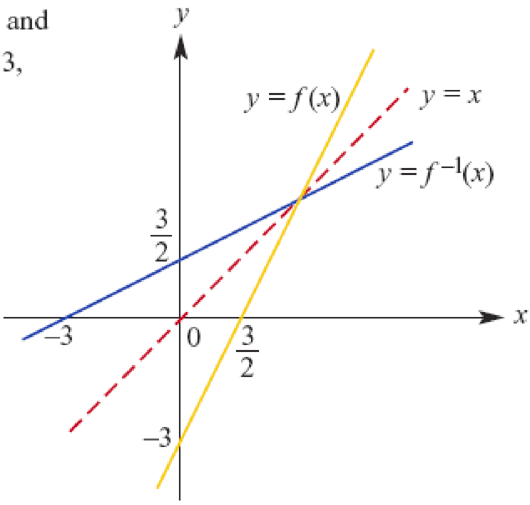
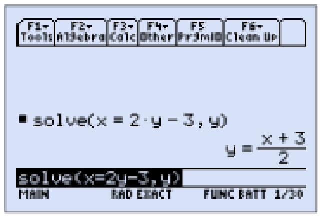



## Using a CAS calculator

Use **1:solve** to find the inverse of the function with rule $f(x) = 2x - 3$.




**Original location: Chapter 6 Example 16 (p.168-169)**



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## Example: Combinations of transformations

Find the equation of the image of $y = \sqrt{x}$ under:

**a** a dilation of factor 2 from the $x$-axis followed by a reflection in the $x$-axis

**b** a dilation of factor 2 from the $x$-axis followed by a translation of 2 units in the positive direction of the $x$-axis and 3 units in the negative direction of the $y$-axis

### Solution

**a** From the discussion above, $(x, y) \rightarrow (x, 2y) \rightarrow (x, -2y)$. Hence if $(x', y')$ is the image of $(x, y)$ under this map, $x' = x$ and $y' = -2y$. Hence $x = x'$ and $y = \frac{y'}{-2}$. The graph of the image will have equation $\frac{y'}{-2} = \sqrt{x'}$ and hence $y' = -2\sqrt{x'}$.

**b** From the discussion above, $(x, y) \rightarrow (x, 2y) \rightarrow (x + 2, 2y - 3)$. Hence if $(x', y')$ is the image of $(x, y)$ under this map, $x' = x + 2$ and $y' = 2y - 3$. Hence $x = x' - 2$ and $y = \frac{y' + 3}{2}$. Thus the graph of the image will have equation $\frac{y' + 3}{2} = \sqrt{x' - 2}$ or $y' = 2\sqrt{x' - 2} - 3$.

## Using a CAS calculator

Press F4, select **1:Define**, and then complete as shown.