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CHAPTER

# 21

MODULE 4

# Applications of financial mathematics

- How do we calculate income tax?
- What do we mean by capital gains tax, stamp duty, GST?
- How do we calculate the interest earned on our bank account balance?
- What is hire purchase?
- How do we calculate and compare a flat rate of interest andan effective rate of interest ?
- What is inflation and how do we calculate its impact?
- How do we calculate and compare flat rate depreciation, reducing balance depreciation and unit cost depreciation?
- How do we calculate the balance of a loan when periodic payments are made?
- How do we calculate the amount of interest paid on a reducing balance loan?
- How do we determine the number of payments required to reach a specified value of a reducing balance loan?
- How do we compare the total cost of loans and investments under a variety of conditions?
- What is a perpetuity?



## 21.1 Percentage changes and charges

**Tax** is a term generally applied to revenue collected by the government. There are various sources of this revenue, many of which you will come across in the future. The amount of each of these taxes is usually calculated as a percentage of the transaction involved. Some of the more common transactions are discussed in this section. Note that in financial matters we are generally concerned with the **financial year**, which is from 1 July of one year until 30 June of the following year.

## Income tax

When you are paid a salary, an amount will be deducted by your employer and sent to the government to pay for facilities such as hospitals, schools and roads. This is called **income tax**, and the amount you pay depends on how much you earn. This amount is not a set percentage, but increases as the amount you earn increases, according to a classification of **tax subdivisions**, or **tax brackets**. The tax rates for each of the subdivisions are called **marginal rates** and the values of these for the financial year 2005–2006 are as shown.

| | *Tax subdivision* ($) | (%) *tax payable (marginal rate)* |
|---|---|---|
| 1 | 0–6000 | 0 |
| 2 | 6 001–21 600 | 17 |
| 3 | 21 601–70 000 | 30 |
| 4 | 70 001–125 000 | 42 |
| 5 | 125 001+ | 47 |

We can use this table to calculate the income tax payable on any salary. The amount you are paid before the income tax is deducted is called your **gross** salary, and the amount after the income tax is deducted is called your **net** salary.

### Example 1 Calculating income tax

John's gross salary is $45 730 per year.

**a** How much does he pay in income tax?

**b** If he is given a raise of $5000 per year, how much of this will he actually receive?

### Solution

**a** **1** Determine the percentage tax payable on each subdivision of John's salary.

Tax on first \$6000 = 0

Tax on \$6001 − \$21 600 = $15\,600 \times \frac{17}{100}$ = \$2652

Tax on \$21 601 − \$45 730 = $24\,130 \times \frac{30}{100}$ = \$7239

**2** Add tax payable at each subdivision to find the total tax payable.

Total tax = 0 + 2652 + 7239 = \$9891

**b** John's new salary is \$50 730, so the whole \$5000 raise will be in subdivision 3 where it is taxed at a marginal rate of 30%.

Tax on John's raise = $5000 \times \frac{30}{100}$ = \$1500

Amount received = 5000 − 1500 = \$3500

## Capital gains tax

Capital gains tax (CGT) is the tax paid on any profit you make from an investment. In that sense, it is a component of your income tax. The profit made from an investment is called a **capital gain**, and you are taxed on this at the appropriate marginal rate (various other conditions may have to be taken into account).

**Example 2** **Calculating capital gains tax**

Daisy buys shares for \$35 000 in October and sells them for \$63 000 in March of the next year. If her salary for that financial year is \$82 390, how much capital gains tax will she pay on the profit she makes on the shares?

**Solution**

1 Firstly we need to find the profit made on the shares.

$$\text{Profit} = \text{selling price} - \text{purchase price}$$
$$= 63\,000 - 35\,000$$
$$= \$28\,000$$

2 Daisy earns \$82 390, so she is in tax subdivision 4, and her marginal tax rate is 42%.

$$\text{Capital gains tax} = 28\,000 \times \frac{42}{100}$$
$$= \$11\,760$$

## Goods and services tax

The goods and services tax (GST) is a tax of 10% which is added to the price of most goods (including cars) and services (such as insurance).

**Example 3** **Calculating GST**

**a** If the cost of electricity supplied in one quarter is \$288.50, how much GST will be added to the bill?

**b** If the selling price of a washing machine is \$990, how much of this is GST?

**Solution**

**a** GST of 10% will be added.

$$\text{GST} = 288.50 \times \frac{10}{100} = \$28.85$$

**b** 1 This is an application of the problems discussed in section 20.1. There we found that when an $r$% increase has been applied:

$$\text{original price} = \text{new price} \times \frac{100}{(100 + r)}$$

We can use this rule here to find the price without GST.

$$\text{Price without GST} = 990 \times \frac{100}{110}$$
$$= 900$$

2 The amount of the GST is the difference between the price without GST and the selling price.

$$\text{GST} = 990 - 900 = \$90$$

## Stamp duty

Stamp duty is a term commonly used to describe the duty or tax charged by the government on various commercial transactions. This includes the **purchase** of land, houses, motor vehicles, life insurance and livestock. Currently stamp duty on **property** is payable according to the table.

| | *Purchase price of property* | *Stamp duty payable* |
|---|---|---|
| 1 | 0–20 000 | 1.4% |
| 2 | 20 001–115 000 | \$280 plus 2.4% of the value in excess of 20 000 |
| 3 | 115 001–870 000 | \$2560 plus 6% of the value in excess of \$115 000 |
| 4 | More than 870 000 | 5.5% of the value |

**Example 4** **Calculating stamp duty on property**

Brett buys a house for \$480 000. What is the value of the stamp duty payable on this purchase?

**Solution**

Since the purchase price is in subdivision 3, this is the rule used to determine stamp duty.

$$\text{Stamp duty} = 2560 \text{ plus } 6\% \text{ of } (480\,000 - 115\,000)$$
$$= 2560 + \frac{6}{100} \times 365\,000$$
$$= \$24\,460$$

## Exercise 21A

*Unless otherwise instructed give your answers to the nearest cent*

### Income tax

*Use the income tax table on page 507 to complete these exercises*

1 Determine the income tax payable on the following gross annual salaries:

a \$5350 b \$23 870 c \$58 690 d \$128 950

2 Maria's gross salary is \$48 925 per year.

a How much does she pay in income tax?

b If she is given a raise of \$4500 per year, how much of this will she actually receive?

3 Edwina's gross annual salary is \$45 732, which she is paid in 12 equal monthly payments.

a What is her gross monthly salary?

b What is her monthly salary after the income tax has been deducted?

4 Michael is paid an hourly rate of \$22.50 in his part-time job.

a If he works 18 hours per week for 52 weeks one year, how much did he earn that year?

b How much tax did he pay in that year? c What net hourly rate was he paid that year?

## Capital gains tax

**5** Millie buys a painting for $13 500 and sells it within the same financial year for $23 000.

**a** How much profit does she make on the painting?

**b** If her other salary for that financial year is $73 390, how much capital gains tax will she pay on the profit she makes on the painting?

**6** Angie's share tradings have given her a profit of $34 345 in one financial year. If her other income for that financial year is $88 390, how much capital gains tax will she pay on the profit she makes on the shares?

**7** Wendy and Frank buy and sell a block of land within the same financial year, making a profit of $62 000.

**a** If Wendy has other income of $132 000, and the profit is all hers, how much capital gains tax must be paid on the land?

**b** If Frank has other income of $32 000, and the profit is all his, how much capital gains tax must be paid on the land?

## GST

**8** Find the GST payable on each of the following (give your answer correct to the nearest cent):

**a** a gas bill of $121.30
**b** a telephone bill of $67.55
**c** a television set costing $985.50
**d** gardening services of $395

**9** The following prices are without GST. Find the price after GST has been added:

**a** a dress worth $139
**b** a bedroom suite worth $2678
**c** a home video system worth $9850
**d** painting services of $1395

**10** If a computer is advertised for $2399 including GST, how much would the computer have cost without GST?

**11** What is the amount of the GST that has been added if the price of a car is advertised as $39 990 including GST?

**12** The telephone bill is $318.97 after GST is added.

**a** What was the price before GST was added?
**b** How much GST must be paid?

## Stamp duty

*Use the table of stamp duty payable on page 509 to complete these exercises*

**13** Tony and Bev buy a house for $660 000. How much stamp duty is payable on this purchase?

**14** The purchase price of a property is $1 280 000. How much stamp duty is payable on this purchase?



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**15** A property is sold for $114 600. What is the value of the stamp duty payable on this purchase?

**16** Louise buys a home unit for $523 000. What is the value of the stamp duty payable on this home unit?

## 21.2 Bank account balances

When we keep money in the bank, we are paid interest. The amount of interest paid depends on two things:

- the rate of interest the bank is paying
- the amount on which the interest is calculated

Often, banks will pay interest on the 'minimum monthly balance', that is, the lowest amount the account contains in each calendar month. When this principle is used we will assume that all months are of equal length, as illustrated in the following example.

**Example 5** **Finding the interest payable**

The table shows the entries in Tom's bank book. If the bank pays interest at a rate of 3.75% per annum on the minimum monthly balance, find the interest payable for the month of June.

| *Date* | *Debit (dollars)* | *Credit (dollars)* | *Total (dollars)* |
|---|---|---|---|
| 30 May | | 10.00 | 900.00 |
| 15 June | 200.000 | | 700.00 |
| 28 June | | 100.00 | 800.00 |
| 3 July | | 50.00 | 850.00 |

**Solution**

**1** Determine the minimum monthly balance for June.

The minimum balance in the account for June was $700.00

**2** Determine the interest payable in June.

$$I = \frac{Prt}{100} = 700 \times \frac{3.75}{100} \times \frac{1}{12} = \$2.1875$$

To the nearest cent, the interest payable is $2.19.

Banks may also offer accounts that pay interest on the daily balance. When interest is paid on the *minimum* daily balance then you should note that when money is **deposited** the daily balance used for the interest calculation is the amount in the account before the deposit was made, but when money is **withdrawn** then the daily balance used for the interest calculation is the amount in the account after the withdrawal. When interest is payable on the minimum daily balance we will include the exact number of days in each month in the calculations, as shown in the following example.

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**Example 6** **Finding the interest earned**

Andrea receives a statement from the bank which gives the detail of her investment account from 1 July until 31 December, 2005. The details are as shown.

| *Date* | *Debit ($)* | *Credit ($)* | *Total($)* |
|---|---|---|---|
| Opening Balance: | | | |
| 1 July | | | 2000.00 |
| 8 August | | 360.00 | 2360.00 |
| 10 September | | 1363.40 | 3723.40 |
| Closing Balance: | | | |
| 31 December | | | 3723.40 |

How much interest has been earned on this account if the bank pays simple interest of 4.5% per annum on the minimum daily balance?

**Solution**

1 The balance from 1 July until 8 August is $2000. How many days is this?

1 July until 8 August $= 31 + 8 = 39$ days

2 Determine the interest payable for this period.

$$\text{Interest} = 2000 \times \frac{4.5}{100} \times \frac{39}{365} = 9.616$$

3 The balance from 9 August until 10 September is $2360. How many days is this?

9 August until 10 September $= 23 + 10 = 33$ days

4 Determine the interest payable for this period.

$$\text{Interest} = 2360 \times \frac{4.5}{100} \times \frac{33}{365} = 9.602$$

5 The balance from 11 September until 31 December is $3723.40. How many days is this?

11 September until 31 December $= 20 + 31 + 30 + 31 = 112$ days

6 Determine the interest payable for this period.

$$\text{Interest} = 3723.40 \times \frac{4.5}{100} \times \frac{112}{365} = 51.414$$

7 Determine the total interest

Total interest $= 9.616 + 9.602 + 51.414 = 70.632$

Thus the total interest earned is $70.63.

## Exercise 21B

1 An account at a bank is paid interest of 0.75% per month on the minimum monthly balance, credited to the account at beginning of the next month. During a particular month the following transactions took place:

| | |
|---|---|
| 7 September | $1500 withdrawn |
| 12 September | $950 withdrawn |

If the opening balance for September was $5595:

**a** What was the balance of the account at the end of September?

**b** How much interest was paid for the month?

**2** The minimum monthly balances for four consecutive months are: $130.00, $460.50, $545.63, $391.49.
How much interest is earned over the 4-month period if it is calculated on the minimum monthly balance at a rate of 4.25% per annum?

**3** An account is opened with an initial deposit of $5525 on 1 April. On 24 April $500 is withdrawn. On 26 May a deposit of $175 is made. If no other transactions are made, find the amount of interest earned from 1 April until 30 June, if the account pays 6% per annum on the minimum monthly balance.

**4** The bank statement opposite shows transactions over a one-month period for a savings account that earns simple interest at a rate of 3% per annum calculated daily and paid monthly.

| *Date* | *Transaction* | *Debit* | *Credit* | *Balance* |
|---|---|---|---|---|
| 1 May | | | | 650.72 |
| 6 May | Coles 447375 | 204.90 | | 445.82 |
| 19 May | Salary | | 795.55 | 1241.37 |
| 1 June | | | | 1241.37 |

**a** How much interest is earned in this month?

**b** If interest is calculated on a minimum monthly balance instead, how much is earned this month?

**5** The bank statement opposite shows transactions over a 1-month period for a savings account that earns simple interest at a rate of 3.75% per annum calculated daily and paid quarterly.

| *Date* | *Transaction* | *Debit* | *Credit* | *Balance* |
|---|---|---|---|---|
| 1 March | | | | 650.72 |
| 8 March | Cash | | 250.00 | 900.72 |
| 21 March | Cash | | 250.00 | 1150.72 |
| 1 April | | | | 1150.72 |

**a** How much interest is earned in March?

**b** If interest is calculated on a minimum monthly balance instead, how much is earned in March?

**6** Michael's account shows the transaction history for the months of July, August and September one year. His account is with a building society, which pays 5.3% simple interest per annum calculated daily. A competitor offers a rate of 5.4% per annum calculated on the minimum monthly balance. Over these three months:

| *Date* | *Debit* | *Credit* | *Balance* |
|---|---|---|---|
| 1 July | | | $73 226 |
| 10 July | $2000 | | $71 226 |
| 29 July | | $4000 | $75 226 |
| 1 Oct | | | $75 226 |

**a** what interest would be earned with the current building society?

**b** what interest would be earned with the competitor?

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7 The details of an investment account are as shown.

How much interest has been earned on this account if the bank pays simple interest of 3% per annum on the minimum daily balance?

| Date | Debit ($) | Credit ($) | Total ($) |
|---|---|---|---|
| Opening Balance: | | | |
| 1 July | | | 500.00 |
| 8 August | | 400.00 | 900.00 |
| 10 December | | 350.00 | 1250.00 |
| Closing Balance: | | | |
| 31 December | | | 1250.00 |

8 Andrew receives a statement from the bank which gives the detail of his investment account from 1 November until 31 December. The details are as shown.

How much interest has been earned on this account if the bank pays simple interest of 4% per annum on the minimum daily balance?

| Date | Debit ($) | Credit ($) | Total ($) |
|---|---|---|---|
| Opening Balance: | | | |
| 1 November | | | 10 000.00 |
| 12 November | | 4350.98 | 14 350.98 |
| 11 December | 2277.44 | | 12 073.54 |
| Closing Balance: | | | |
| 31 December | | | |

## 21.3 Hire purchase

Instead of saving for the purchase of an item, an option is to enter into a hire-purchase agreement. This means the purchaser agrees to hire the item from the vendor and make periodical payments at an agreed rate of interest. At the end of the period of the agreement, the item is owned by the purchaser. If the purchaser stops making payments at any stage of the agreement, the item is returned to the vendor and no money is refunded to the purchaser.

The interest rate being charged in these contracts is not always stated explicitly. There are two different interest rates that could be stated, or determined, and it is important to distinguish between them so that we can judge just how much we are paying for an item. These are the flat rate of interest $r_f$ and the effective rate of interest $r_e$.

### Flat rate of interest

The interest paid given as a percentage of the original amount owed is called the **flat rate of interest**. The flat interest rate is exactly the same as the simple interest rate, but is generally called by this name in the hire-purchase context. In Chapter 20 we established that for simple interest:

$$r = \frac{100I}{Pt}$$

where $P$ is the principal, $I$ is the amount of interest, $t$ is the time in years.



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In the case of hire purchase, we will define the formula as follows:

Flat interest rate per annum, $r_f = \dfrac{100I}{Pt}$

Where $I$ = total interest paid

$P$ = principal owing after the deposit has been deducted

$t$ = the number of years

## Example 7 Calculating the interest and the flat rate of interest when $t = 1$

Nicole enters into a hire-purchase agreement in order to purchase a car: The price of the car is \$10 000. She agrees to pay a deposit of \$2000, and then make 12 payments of \$800 over the remainder of the year.

**a** How much interest has she paid?

**b** What is the flat rate of interest that this represents?

### Solution

**a** **1** First we need to determine how much Nicole has paid in total.

Total paid = deposit + repayments
= 2000 + 12 × 800
= \$11 600

**2** Determine the interest paid. This is the difference between the purchase price and the total paid.

Interest paid = 11 600 − 10 000
= \$1600

**b** **1** Apply the appropriate formula with $P = 8000$ (since only \$8000 is owing after the deposit is paid), $I = 1600$ and $t = 1$ (since the car is paid off in one year) to find the value of $r_f$.

$$r_f = \frac{100I}{Pt} = \frac{100 \times 1600}{8000 \times 1} = 20$$

**2** Since the unit of time is years, the interest rate can be written as the interest per annum.

Interest rate = 20% per annum

## Example 8 Calculating the interest and the flat rate of interest when $t \neq 1$

A hire-purchase contract for a lounge suite requires Rob to pay a deposit of \$500 and then make 6 monthly payments of \$345. If the price of the lounge suite is \$2300:

**a** how much interest will he pay?

**b** what is the flat rate of interest per annum that this represents?



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**Solution**

**a 1** Determine how much Rob has paid in total.

Total paid = deposit + repayments
= 500 + 6 × 345
= \$2570

**2** Determine the interest paid, the difference between the total paid and the purchase price.

Interest paid = 2570 − 2300
= \$270

**b 1** Apply the appropriate formula with $P = 1800$ (since only \$1800 is owing after the deposit is paid), $I = 270$ and $t = 0.5$ (since the lounge suite is paid off in six months) to find the value of $r_f$.

$$r_f = \frac{100I}{Pt} = \frac{100 \times 270}{1800 \times 0.5} = 30$$

**2** Since the unit of time is years, the interest rate can be written as the interest per annum.

Interest rate = 30% per annum

## Effective interest rate

The flat rate of interest implied by a hire-purchase contract can be quite misleading. Consider Example 8 again. Rob is paying a flat rate of interest of 30%, but he is actually paying a much higher **effective interest rate** than this. This is because he is making periodical repayments throughout the course of the contract and, in fact, for a lot of the time he actually owes quite a lot less than the \$1800 owed at the beginning. To determine the effective interest rate, we work out the **average amount** he owes over the period of the loan, and use this as the denominator for the interest calculation.

At the beginning of the contract Rob owes \$1800, which is to be paid off in six equal monthly instalments. If there were no interest at all, Rob would pay \$300 each month $\left(\frac{1800}{6}\right)$. Since Rob pays \$345 each month, this amount can be considered as \$300 off the principal, and \$45 interest. So, how much principal does Rob actually owe? The table below gives details of the agreement, month by month.

| *Month* | *Principal owed* (\$) | *Principal paid* (\$) | *Interest paid* (\$) | *Total payment* (\$) |
|---|---|---|---|---|
| 1 | 1800 | 300 | 45 | 345 |
| 2 | 1500 | 300 | 45 | 345 |
| 3 | 1200 | 300 | 45 | 345 |
| 4 | 900 | 300 | 45 | 345 |
| 5 | 600 | 300 | 45 | 345 |
| 6 | 300 | 300 | 45 | 345 |
| Total | 6300 | 1800 | 270 | 2070 |

The average amount of principal owed over the term of the contract is thus:

$$\text{average amount owing} = \frac{6300}{6} = 1050$$

and hence:

$$\textbf{effective interest rate} = \frac{100I}{\text{average amount owed} \times t}$$
$$= \frac{100 \times 270}{1050 \times 0.5}$$
$$= 51.4\%$$

which is a lot higher than the flat rate of interest of 30% previously calculated.

The average principal owing can be calculated without listing every term as we have done in the previous table if we recognise that the amount owing each month forms an arithmetic sequence, with the first term equal to the principal owed at the start, $P$ ($1800 in this example), and the common difference equal to the amount paid off the principal each month ($300 in this example).

Putting all of this together leads to a general rule for effective interest rate per annum.

**Effective interest rate**

Effective interest rate per annum, $r_e = \frac{100I}{Pt} \times \frac{2n}{(n+1)}$

Where $I$ = total interest paid
$P$ = principal owing after the deposit has been deducted
$t$ = number of years
$n$ = number of payments made in total

## Example 9 Determining the effective interest rate

Monique arranges to purchase a stereo costing $1690. She agrees to pay a deposit of $150, and 18 equal monthly payments of $95. Find:

**a** the flat rate of interest per annum
**b** the effective rate of interest per annum that she is paying on this contract.

### Solution

**a 1** Determine the total amount paid

Total paid = 150 + 18 × 95 = $1860

**2** The difference between the total paid and the purchase price is the interest paid.

Interest paid = 1860 − 1690 = $170

**3** Apply the flat rate formula with $P = 1540$ (the amount owing after the deposit is paid), $I = 170$ and $t = 1.5$ (the stereo is paid off in eighteen months) to find the value of $r_f$.

$$r_f = \frac{100I}{Pt} = \frac{100 \times 170}{1540 \times 1.5} = 7.36\% \text{ per annum}$$

**b 1** Substitute $P = 1540$, $I = 170$ and $t = 1.5$ and $n = 18$ into the formula for effective interest rate.

$$r_e = \frac{100I}{Pt} \times \frac{2n}{(n+1)} = \frac{100 \times 170}{1540 \times 1.5} \times \frac{2 \times 18}{(18+1)} = 13.94\%$$

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It may also be useful to be able to calculate the effective interest rate from the flat interest rate. Rearranging the expression for effective interest rate readily leads to the following relationship:

**Effective interest rate from flat interest rate**

The effective interest rate per annum can be determined from the flat interest rate per annum using the expression:

$$r_e = r_f \times \frac{2n}{n+1}$$

where $r_e$ = effective interest rate per annum
$r_f$ = flat interest rate per annum
$n$ = number of payments made in total

If $n$ is large, then the expression for $\frac{n}{n+1} \approx 1$. This gives a very quick way of estimating the effective interest rate per annum from the flat interest rate per annum.

If $n$ is large, then:

$$r_e \approx r_f \times 2$$

where $r_e$ = effective interest rate per annum
$r_f$ = flat interest rate per annum

**Example 10** **Determining the effective interest rate from the flat interest rate**

Repayments on Tom's hire-purchase agreement are based on a flat interest rate per annum of 15%, and he is to make 60 repayments. Calculate:

**a** the exact value of the effective interest rate per annum
**b** an approximate value of the effective interest rate per annum

**Solution**

**a** Substitute $r_f = 15$ and $n = 60$ in the formula for effective interest rate.

$$r_e = r_f \times \frac{2n}{n+1} = 15 \times \frac{2 \times 60}{60+1} = 29.5\%$$

**b** Use the approximation rule. Note that since $n$ is large in this example, the answers to **a** and **b** are very similar.

$$r_e \approx r_f \times 2 = 15 \times 2 = 30\%$$

## Exercise 21C

Skillsheet TEACHER

**1** Calculate the flat interest rate per annum on each of the following loans:

**a** $2400, to be repaid in 30 monthly instalments of $90
**b** $1500, to be repaid in 15 monthly instalments of $115

c $8000, to be repaid in 24 monthly instalments of $350
d $750, to be repaid in 6 monthly instalments of $135
e $7250, to be repaid in 18 monthly instalments of $420

2 Calculate the flat interest rate per annum on each of the following loans:

a $2000, to be repaid in 52 weekly instalments of $42.50
b $750, to be repaid in 26 weekly instalments of $29.90
c $5500, to be repaid in weekly instalments of $150 over 9 months
d $6000, to be repaid in weekly instalments of $97.50 over 15 months
e $3000, to be repaid in weekly instalments of $22.70 over 3 years

3 Evaluate the total interest paid, and hence the monthly repayments, on the following contracts:

a $3000 at a flat interest rate of 18.5% per annum over one year
b $5500 at a flat interest rate of 12.5% per annum over 18 months
c $15 000 at a flat interest rate of 14% per annum over 72 months
d $2250 at a flat interest rate of 9.75% per annum over 6 months
e $4200 at a flat interest rate of 11.25% per annum over 15 months

4 Evaluate the total interest paid, and hence the weekly repayments, on the following contracts:

a $1800 at a flat interest rate of 11.3% per annum over 9 months
b $14 950 at a flat interest rate of 9.8% per annum over 2 years
c $15 000 at a flat interest rate of 14.4% per annum over 5 years
d $22 250 at a flat interest rate of 11.75% per annum over 6 months
e $1250 at a flat interest rate of 10.75% per annum over 3 months

5 The following items were purchased on hire-purchase contracts. For each one, calculate the amount of interest to be paid:

| | *Price* | *Deposit* | *Interest rate* | *Length of loan* | *Payment period* |
|---|---|---|---|---|---|
| a Fitness equipment | $350 | $35 | 10% p.a. | 1 year | monthly |
| b CD player | $790 | $100 | 12% p.a. | 15 months | quarterly |
| c Computer | $3550 | $450 | 14.5% p.a. | 2.5 years | monthly |

6 The local electrical store advertises a television set for $889 or $50 deposit and $25 per week for three years.

a How much does the television set end up costing under the hire-purchase scheme?
b What is the flat rate of interest per annum?
c What is the effective rate of interest per annum?

7 A personal loan of $5000 over a period of three years costs $250 per month.

a What is the flat rate of interest per annum?
b What is the effective rate of interest per annum?



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**8** A personal loan of $8000 over a period of five years costs $200 per month.

**a** What is the flat rate of interest per annum?
**b** What is the effective rate of interest per annum?

**9** The cash price of a pair of skis is $775. A deposit of $85 is made and the balance with interest is to be paid over two years in monthly instalments of $35. Find:

**a** the amount of interest charged **b** the flat rate of interest per annum
**c** the effective rate of interest per annum

**10** A car yard offers Jane its latest model at $41 999, less a trade-in of $23 200 for her old car. Alternatively, she can buy the vehicle by paying $325 per month for 6 years (also with the trade-in). Find:

**a** the total amount paid on the hire-purchase plan
**b** the amount saved by Jane if she pays cash
**c** the flat rate of interest per annum on the hire-purchase plan
**d** the effective rate of interest per annum on the hire-purchase plan

**11** Daniel buys a mountain bike on a time payment plan, which specifies a $150 deposit on the $1100 bike and monthly instalments of $48 over two years.

**a** How much interest is Daniel to pay?
**b** What is the flat rate of interest per annum?
**c** What is the effective rate of interest per annum?

**12** A customer borrows $2000 from the bank to purchase a new set of golf clubs. The terms of the loan are a simple interest rate of 18% per annum, taken over a period of 27 months. Find:

**a** the total interest owing **b** the monthly repayment
**c** the effective rate of interest per annum: **i** exactly **ii** approximately

**13** Under a hire-purchase contract for a new computer costing $4560, Janine pays a deposit of $320 and a balance at a simple interest rate of 15% per annum, taken over a period of 24 months. Find:

**a** the total interest paid **b** the monthly repayment
**c** the effective rate of interest per annum: **i** exactly **ii** approximately

## 21.4 Inflation

**Inflation** is a term that describes the continuous upward movement in the general level of prices. This has the effect of steadily **reducing** the **purchasing power** of your money, that is, what you can actually buy with your money.

In the early 1970s, inflation rates were very high, up to figures around 16% and 17%. Inflation in Australia has been relatively low in recent years.

- Since 1970, inflation has averaged 6.8% per year.
- Since 1990, it has averaged 2.1% per year.

The effect of inflation on prices is illustrated in the following example.

**Example 11** **Determining the effect of inflation on prices**

Suppose that inflation is recorded as 2.7% in 2006, and 3.5% in 2007, and that a loaf of bread costs $2.20 at the end of 2005. If the price of bread increases with inflation, what will be the price of the loaf at the end of 2007?

**Solution**

**1** Determine the price of the loaf of bread at the end of 2006 after a 2.7% increase.

$$\text{Increase in price (2006)} = 2.20 \times \frac{2.7}{100} = 0.06$$

**2** Calculate the price at the end of 2006.

$$\text{Price (2006)} = 2.20 + 0.06 = \$2.26$$

**3** Determine the price of the loaf of bread at the end of 2007 after a further 3.5% increase.

$$\text{Increase in price (2007)} = 2.26 \times \frac{3.5}{100} = 0.08$$

**4** Calculate the price at the end of 2006.

$$\text{Price (2007)} = 2.26 + 0.08 = \$2.34$$

While this difference in price seen in Example 11 does not seem a lot, you will be aware from our earlier discussion of compound interest that even if inflation holds steady at a low 2.1% per year for 20 years, prices will still increase a lot as illustrated in the following example.

**Example 12** **Determining the effect of inflation on prices over a longer period**

Suppose that a one-litre carton of milk costs $1.70 today.

**a** What will be the price of the one-litre carton of milk in 20 years' time if the average annual inflation rate is 2.1%?

**b** What will the price of the one-litre carton of milk in 20 years' time if the average annual inflation rate is 6.8%?

**Solution**

**a** **1** This is the equivalent of investing $1.70 at 2.1% interest compounding annually, so we can use the compound interest formula from Section 20.3.

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

**2** Substitute $P = \$1.70$, $t = 20$, $r = 2.1$ in the formula to find the price in 20 years.

$$\text{Price} = 1.70 \times \left(1 + \frac{2.1}{100}\right)^{20} = \$2.58 \text{ to the nearest cent}$$

**b** Substitute $P = 1.70$, $t = 20$, $r = 6.8$ in the formula and evaluate.

$$\text{Price} = 1.70 \times \left(1 + \frac{6.8}{100}\right)^{20} = \$6.34 \text{ to the nearest cent.}$$



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Another way of looking at the effect of inflation on our money is to consider what a sum of money today would buy in the future. That is, to convert projected dollar numbers back into present day values so you can think in today's money values. Suppose, for example, you put \$100 in a box under the bed, and leave it there for 10 years. When you go back to the box, there is still \$100, but the question is, what could you buy with this amount in 10 years' time? To find out we need to 'deflate' this amount back to current-day purchasing power dollars. We can do this using the compound interest formula from Section 20.3:

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

where \$$A$ is the actual amount of money at the end of the time period, \$$P$ is the deflated value of the money, $r$% per annum the average inflation rate over a time period of $t$ years.

Suppose that there has been an average rate of inflation of 4% over the ten year period. Substituting $A = 100,\ r = 4$ and $t = 10$ gives:

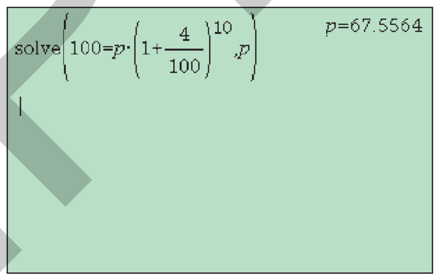
$$100 = P \times \left(1 + \frac{4}{100}\right)^{10}$$

Rearranging gives:

$$P = \frac{100}{(1 + 0.04)^{10}}$$

$$= \$67.56 \text{ to the nearest cent}$$

or use solve:




That is, the money that was worth \$100 at the time it was put away has a purchasing power of only \$67.58 after ten years if the inflation rate has averaged 4% per annum.

## Example 13 Investigating purchasing power

If savings of \$100 000 are hidden in a mattress in 2006, what is the purchasing power of this amount in 8 years' time if the average inflation rate over this period is 3.7%? Give your answer to the nearest dollar.

### Solution

1 Write the compound interest formula with $P$ (the purchasing power, which is unknown), $A = 100\,000$ (current value), $r = 3.7$ and $t = 8$.

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

$$100\,000 = P \times \left(1 + \frac{3.7}{100}\right)^8$$

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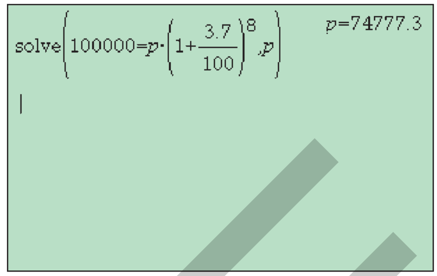
**2** Use your calculator to solve this equation for *P*.




**3** Write your answer.

*The purchasing power of $100 000 in 8 years is $74 777, to the nearest dollar.*

## Exercise 21D

**1** Suppose that inflation is recorded as 2.7% in 2006, and 3.5% in 2007, and that a magazine costs $3.50 at the end of 2005. Assume the price increases with inflation.

**a** What will be the price of the magazine at the end of 2006?
**b** What will be the price of the magazine at the end of 2007?

**2** Suppose that Henry's wage is such that he receives a salary increase at the end of each year equal to the rate of inflation for that year. If inflation is recorded as 3.2% in 2006, and 5.3% in 2007 and Henry's weekly salary is $825 at the end of 2005:

**a** what is Henry's salary at the end of 2006?
**b** what is Henry's salary at the end of 2007?

**3** Suppose that inflation is recorded as 4.5% in 2005, 2.1% is 2006 and 3.3% in 2007, and that a kilo of oranges costs $4.95 at the end of 2004. If the price of oranges increases with inflation, what will be the price per kilo at the end of 2007?

**4** Suppose that inflation is recorded as 4.1% in 2005, 3.1% is 2006 and 2.1% in 2007, and that rental on a certain property is $295 per week at the end of 2004. If the real estate agent increases the rent in line with inflation, what will be the weekly rent at the end of 2007?

**5** Suppose that a loaf of bread costs $2.40 today.

**a** What will be the price of the loaf of bread in 10 years' time if the average annual inflation rate is 2.3%?
**b** What will the price of the bread in 10 years' time if the average annual inflation rate is 6.7%?

**6** Suppose that the cost of petrol per litre is $1.20 today.

**a** What will be the price of petrol per litre in 20 years' time if the average annual inflation rate is 1.9%?
**b** What will be the price of petrol per litre in 20 years' time if the average annual inflation rate is 7.1%?



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**7** Suppose that a particular house is sold at auction for $500 000. If the price of the houses increases with the inflation rate, what will be the price of the house in 12 years' time:

**a** if the average inflation rate over the 12-year period is 2.6%?
**b** if the average inflation rate over the 12-year period is 6.9%?

**8** If a certain car is priced at $52 500, and the price of the car increases with the inflation rate, what will be the price of the car in 5 years' time:

**a** if the average inflation rate over the 5-year period is 3%?
**b** if the average inflation rate over the 5-year period is 14%?

**9** If savings of $200 000 are hidden in a mattress in 2006, what is their purchasing power in 10 years' time:

**a** if the average inflation rate over the 10-year period is 3%?
**b** if the average inflation rate over the 10-year period is 13%?

**10** Suppose Ann-Marie puts $1000 cash in her safe. What is its purchasing power in 20 years' time:

**a** if the average inflation rate over the 20-year period is 2.6%?
**b** if the average inflation rate over the 20-year period is 6.9%?
**c** if the average inflation rate over the 20-year period is 14.3%?

**11** Suppose Ben buries $5000 cash in the garden. What is the purchasing power of this amount in 5 years' time:

**a** if the average inflation rate over the 5-year period is 2.1%?
**b** if the average inflation rate over the 5-year period is 5.7%?
**c** if the average inflation rate over the 5-year period is 12.8%?

## 21.5 Depreciation

People in business need to keep track of the current value of the items used in their business. The value of an item of equipment used in a company is an important factor in determining the value of that company, and in determining whether or not the company is profitable. For example, if the proprietor of a print shop buys a very sophisticated photocopier costing $500 000, and it only lasts five years, then the income generated by the photocopier needs to exceed the replacement value of the photocopier before the business can be considered profitable.

Most equipment will decrease in value over time – it is said to **depreciate** in value. The amount by which the value of an item decreases is called the **depreciation**, and the new reduced value of the item is known as its **book value**. Once the item has reached the stage where it can no longer be used profitably by the company, it is sold off. The price that the item is expected to fetch at this point is called the **scrap value** of the item.

There are three main methods used to determine the depreciated value of an item: unit cost, flat rate and reducing balance.

## Unit cost depreciation

Some items of equipment are valued not by their age but by the amount of use they have had. For example, two company cars could be the same age, but if one has travelled 50 000 km, and the other 200 000 km, then they are not worth the same amount to the company. The company may value these cars on the basis of how many kilometres they have done, as this has implications on how many more kilometres they are able to do. This method is called **unit cost depreciation**.

### Example 14 Determining unit cost depreciation

A machine originally costing $75 000 is expected to produce 200 000 CDs. The output of the machine in each of the first three years was 18 000, 17 000 and 19 000 units respectively. If the anticipated scrap value of the machine is $ 10 000, find:

**a** its book value at the end of each of the three years

**b** the number of years the machine might be expected to last if the average production per year for the remainder of its life is 18 000 CDs

### Solution

**a 1** Work out the unit cost by dividing the purchase price by the production.

$$\text{Unit cost} = \frac{75\,000 - 10\,000}{200\,000} = \$0.325$$

**2** The book value at the end of the first year is the purchase price minus the cost for 18 000 units.

Book value at the end of year 1

$$= 75\,000 - 18\,000 \times 0.325$$
$$= \$69\,150$$

**3** The book value at the end of the second year is the book value at the end of the first year minus the cost for 17 000 units.

Book value at the end of year 2

$$= 69\,150 - 17\,000 \times 0.325$$
$$= \$63\,625$$

**4** The book value at the end of the third year is the book value at the end of the second year minus the cost for 19000 units.

Book value at the end of year 3

$$= 63\,525 - 19\,000 \times 0.325$$
$$= \$57\,450$$

**b 1** Calculate the average depreciation per year if the production is 18 000 units.

$$\text{Average depreciation} = 18\,000 \times 0.325 = \$5850 \text{ per year}$$

**2** Calculate how much more the machine can be depreciated before it will be scrapped.

$$\text{Further depreciation} = 57\,450 - 10\,000 = \$47\,450$$

**3** Calculate how many years this will be at an average rate of $5850 per year.

$$\text{Remaining number of years} = \frac{47\,450}{5850} = 8.1$$

**4** Calculate the total number of years the machine will be in use.

Thus, the total number of years the machine will be in use $= 3 + 8.1$
$= 11.1$ years

From this we can write some general rules for unit cost depreciation. To calculate unit cost:

**Unit cost**

$$\text{unit cost} = \frac{\text{purchase price} - \text{scrap value}}{\text{total production}}$$

To calculate the depreciation, and hence the book value of the item:

**Depreciation and book value**

$$\text{depreciation } (D) = \text{unit cost} \times n$$

$$\text{book value } (V) = \text{purchase price} - \text{unit cost} \times n$$

where $n =$ the number of units produced

$V =$ the book value after the production of $n$ units

To determine the length of time the item will be in use:

**Length of time in use**

$$\text{length of time item will be in use } (t) = \frac{\text{purchase price} - \text{scrap value}}{\text{average depreciation per year} \times \text{unit cost}}$$

## Flat rate depreciation

Flat rate depreciation is when the value of the item is reduced by the same percentage of the purchase price (or amount) for each year it is in use. It is the equivalent, but opposite, situation to simple interest, where an investment increases by a constant amount each year. We have already established formulas for the value of the simple interest and the amount of a loan or investment in Section 20.2. These formulas can also be applied to the calculation of flat rate depreciation.

**Flat rate depreciation**

$$D = \frac{\text{purchase price} \times \text{interest rate (per annum)} \times \text{length of time (in years)}}{100}$$

$$= \frac{Prt}{100}$$

where $D$ is the depreciation of the item after $t$ years, $P$ is the purchase price of the item, $r$ is the flat rate of depreciation and $t$ is the time (in years).

To determine the book value of the item:

**Book value**

$$V = \text{purchase price} - \text{depreciation}$$
$$= P - \frac{Prt}{100}$$

where $V$ is the book value of the item after $t$ years, $P$ is the purchase price of the item, $r$ is the flat rate of depreciation and $t$ is the time (in years).

**Example 15** **Determining flat rate depreciation and book value**

A computer system costs $9500 to buy, and decreases in value by 10% of the purchase price each year.

**a** What is the amount of the depreciation after 4 years?

**b** Find its book value after 4 years.

**Solution**

**a** Substitute $P = 9500$, $r = 10$ and $t = 4$ in the formula for flat rate depreciation.

$$D = \frac{Prt}{100} = \frac{9500 \times 10 \times 4}{100} = \$3800$$

**b** The book value at the end of the 4 years is the cost of the item less the depreciation.

Book value at the end of year 4:

$$V = 9500 - 3800 = \$5700$$

The relationship between the age of the item and its book value at the end of each year can also be shown graphically. As in the case of simple interest, the relationship between book value and age for flat rate depreciation is linear. For this reason, this form of depreciation is also sometimes called **straight line depreciation**.

The graphic calculator can be set up to generate a table for flat rate depreciation over time.

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## How to determine flat rate depreciation and book value using the TI-Nspire CAS

A computer system costs \$9500 to buy and decreases in value by 10% of the purchase price each year.

**a** What is the amount of depreciation after 4 years?

**b** Find its book value after 4 years.

**Steps**

**1** Write expressions for depreciation and book value.

$$\text{depreciation} = \frac{9500 \times 10 \times t}{100}$$

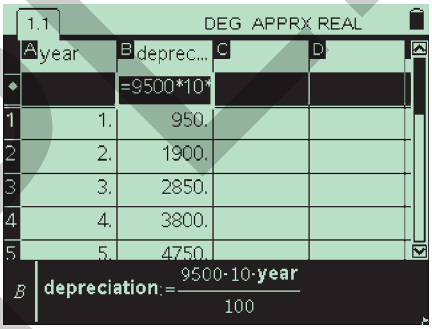
$$\text{book value} = P - \frac{Prt}{100} = 9500 - \frac{9500 \times 10 \times t}{100}$$

**2** Start a new document by pressing (ctrl) + (N), and select **3:Add Lists & Spreadsheet**.

**a** Name three lists: *year* (for *t*), *depreciation*, and *book_value*, respectively.
*Hint*: Use (ctrl) + (⊖) for the underscore or just type *bookvalue*.

**b** Enter the numbers **1, 2, 3, . . ., 10** into the list *year*.

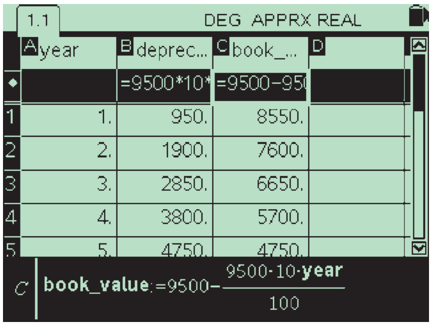
**c** Move the cursor to the grey formula cell of the list *depreciation* and type **=(9500 × 10 × *year*)/100** Press (enter) to calculate the values for depreciation.




**3** Move the cursor to the grey formula cell of the list *book_value* and type

**=9500 − (9500 × 10 × *year*)/100**

Press (enter) to calculate the values for book value.




**Notes:**

**1** An alternative formula to use to calculate the list *book_value* would be

**=9500− depreciation**

**2** You can use the (sto→ var) key to display the variable list rather than retyping in the list names.

**4** Scrolling through the table we see that after 4 years the depreciation is \$3800 and the book value is \$5700.

## How to determine flat rate depreciation and book value using the ClassPad

A computer system costs $9500 to buy and decreases in value by 10% of the purchase price each year.

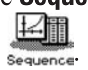
**a** What is the amount of depreciation after 4 years?

**b** Find its book value after 4 years.

### Steps

**1** Write expressions for depreciation, $D$, and book value, $V$.

$$D = \frac{Prt}{100} = \frac{9500 \times 10 \times t}{100}$$
$$= 950t$$
$$V = P - \frac{Prt}{100} = 9500 - \frac{9500 \times 10 \times t}{100}$$
$$= 9500 - 950t$$

**2** From the application menu, locate and open the **Sequence** application, 



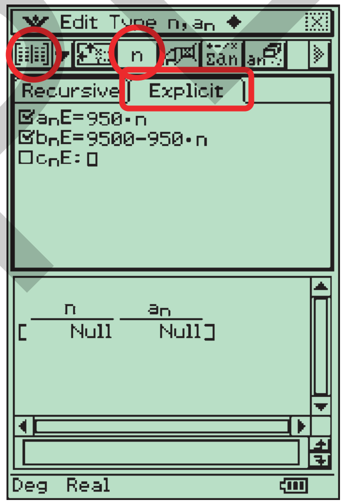
Select the **Explicit** tab and then adjacent to

- $a_n$**E:** enter the rule for depreciation **950*n***
- $b_n$**E**: enter the rule for book **9500 − 950*n***

Press EXE after entering each expression.

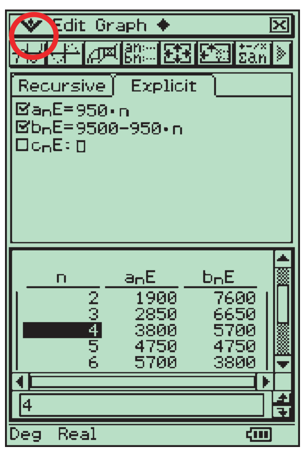
**Note:** Use the ***n*** that is found in the toolbar ( n ).

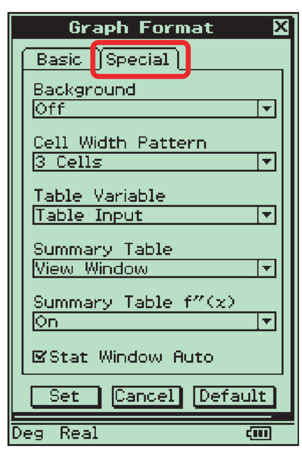
Finally, tap from the toolbar to view a table of values.




**3** Scrolling down the table, it can be seen that after 4 years the depreciation is $3800 and the book value is $5700.

**Note:** To display the list values for both functions, tap the settings ( ) icon, then select the **Special** tab and confirm the **Cell Width Pattern** is set at **3 Cells**.



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As with simple interest, the formulas for flat rate depreciation and book value can be rearranged to find the value of any one of the variables, when the values of other variables are known.

## Reducing balance depreciation

Reducing balance depreciation is when the value of the item is reduced by a constant percentage of its value in the preceding year for each year it is in use. It is the equivalent, but opposite, situation to compound interest, where an investment increases by a constant percentage each year. We can apply some of our knowledge of compound interest from Section 20.3 to this situation. Earlier we established that the amount of money ($\$A$) that would result from investing $\$P$ at $r\%$ per annum compounded annually for a time period of $t$ years is:

$$A = P \times \left(1 + \frac{r}{100}\right)^t$$

Extending this to reducing balance depreciation we can say:

The book value of an item ($\$V$), which has a purchase price of $\$P$, and which is depreciating at a rate of $r\%$ per annum compounded annually for a time period of $t$ years is:

$$V = P \times \left(1 - \frac{r}{100}\right)^t$$

and the amount of the deprecation is given by

$$D = P - V$$

Note that the sign is now negative because the value of the item is now decreasing.

**Example 16** **Reducing balance depreciation**

A computer system costs \$9500 to buy, and decreases in value by 20% each year.

**a** What is the book value of the computer after 4 years?

**b** By how much has the value of the computer depreciated over the 4 years.

**Solution**

**a** Substitute $P = 9500$, $r = 20$ and $t = 4$ in the formula for book value with reducing balance depreciation.

$$V = P \times \left(1 - \frac{r}{100}\right)^t$$

$$= 9500 \times \left(1 - \frac{20}{100}\right)^4 = \$3891.20$$

**b** The amount of the depreciation over 4 years is the cost of the computer less the book value.

Depreciation at the end of year 4:

$$D = 9500 - 3891.20 = \$5608.80$$

The relationship between the age of the item and its book value at the end of each year can also be shown graphically. As in the case of compound interest, the relationship between book value and age for reducing balance depreciation is nonlinear.

The graphics calculator can be set up to generate a table for reducing balance depreciation over time.

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## How to determine reducing balance depreciation and book value using the TI-Nspire CAS

A computer system costs $9500 to buy and decreases in value by 20% each year.

**a** What is the book value of the computer after 4 years?

**b** By how much has the value of the computer depreciated over the 4 years?

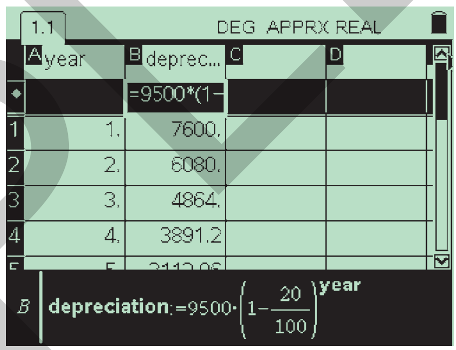
### Steps

**1** Write expressions for book value and depreciation.

$$\text{book value} = 9500 \times \left(1 - \frac{20}{100}\right)^t$$

$$\text{depreciation} = 9500 - 9500 \times \left(1 - \frac{20}{100}\right)^t$$

**2** Start a new document by pressing ctrl + N, and select **3:Add Lists & Spreadsheet**.

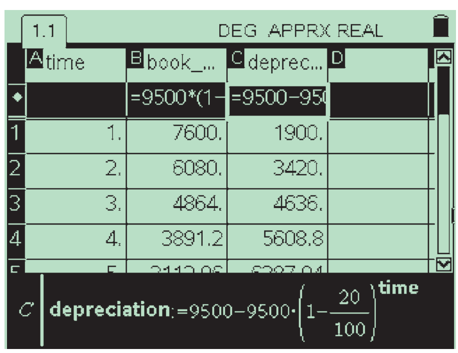



**a** Name three lists, *year* (*t*), *book_value* and *depreciation*, respectively.
*Hint*: Use ctrl + ⊖ for the underscore or just write as *bookvalue*.

**b** Enter the numbers **1**, **2**, **3**, . . ., **10** into the list *year*.

**c** Move the cursor to the grey formula cell of the list *book_value* and type **= (9500 × (1 − 20/100)*^year^*)**
Press enter to calculate the values for book value.

**3** Move the cursor to the grey formula cell of the list *depreciation* and type **= 9500 − (9500 × (1 − 20/100)*^year^*)**
Press enter to calculate the values for depreciation.
**Note:** You can use the sto→var key to display the variable list rather than retyping in the list names.




**4** Scrolling through the table we see that after 4 years the book value is $3891.20 and the depreciation is $5608.80.

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## How to determine reducing balance depreciation and book value using the ClassPad

A computer system costs \$9500 to buy and decreases in value by 20% each year.

**a** What is the book value of the computer after 4 years?

**b** By how much has the value of the computer depreciated over the 4 years?

### Steps

**1** Write expressions for depreciation, $D$, and book value, $V$.

Note that the depreciation is the cost of the computer less the book value.

$$V = 9500 \times \left(1 - \frac{20}{100}\right)^t$$

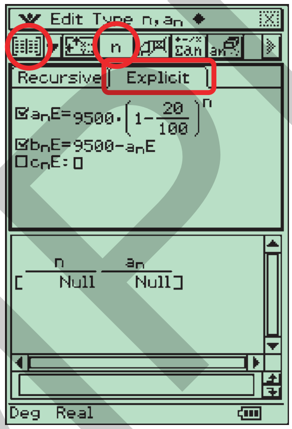
$$D = 9500 - V$$

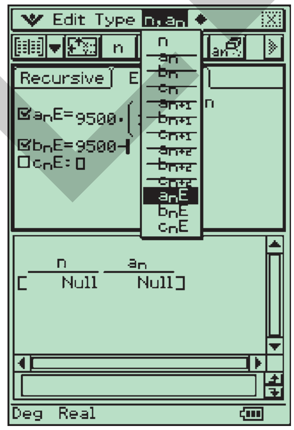
**2** From the application menu, locate and open the **Sequence** application.

Select the **Explicit** tab and then adjacent to

- $a_n$**E**: enter the rule for book value $9500 \times (1 - 20/100)^n$
- $b_n$**E**: enter the rule for depreciation $9500 - a_nE$

Press (EXE) after entering each equation.

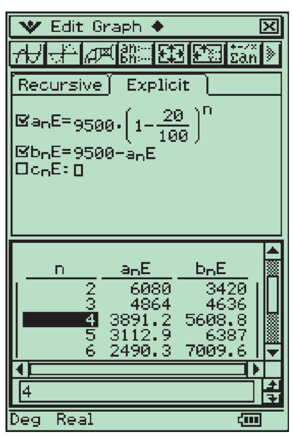



**Note:** Tap $n$, $a_n$ in the menu bar and then select $a_n$**E** (the depreciation value) to enter it into the equation $b_n$**E**.

Finally, tap [table icon] from the toolbar to view a table of values.

**3** Scrolling down the table, it can be seen that after 4 years the book value is \$3891.20 and the depreciation is \$5608.80, as calculated previously.



## Exercise 21E

1 A machine originally costing $37 000 is expected to produce 100 000 units. The output of the machine in each of the first three years was 5234, 6286 and 3987 units respectively. Its anticipated scrap value is $5000.

- a What is the unit cost for this machine?
- b Find the total production over the first three years, and hence the book value at the end of three years.
- c Estimate how many years it will be in use, if the average production during its life is 5169 units per year.

2 A company buys a taxi for $29 000. It depreciates at a rate of 25 cents per kilometre. If the taxi has a scrap value of $5000, find how many kilometres it will have travelled by the time it reaches its scrap value.

3 If a car is valued at $35 400 at the start of the year, and at $25 700 at the end of the year, what has been the unit cost per kilometre if it travelled 25 000 km that year?

4 A printing machine costing $110 000 has a scrap value of $2500 after it has printed 4 million pages.

- a Find:
  - i the unit cost of the machine
  - ii the book value of the machine after printing 1.5 million pages
  - iii the annual depreciation charge of the machine if it prints 750 000 pages per year
- b Find the book value of the printing machine after five years if it prints on average 750 000 pages per year.
- c How many pages has the machine printed by the time the book value is $70 000, if it prints on average 750 000 pages per year?

5 A sewing machine originally cost $1700 and decreases in value by 12.5% of the purchase price per year.

- a What is the amount of the depreciation after 3 years?
- b Find its book value after 3 years.

6 A harvester was bought for $65 000 and it decreases by 10% of the purchase price per annum.

- a Write down the rule relating book value, flat rate of depreciation and time in years.
- b Use your graphic calculator to draw a graph of book value against time in years.
- c Find the amount of the yearly depreciation.
- d If the scrap value of the harvester is $13 000, for how many years will it be in use?

**7** A computer depreciates at a flat rate of 22.5% of the purchase price per annum. Its purchase price is $5600.

**a** What is the book value of the computer after 3 years?
**b** After how long will the computer be written off if the scrap value is nil?

**8** A machine costs $7000 new and depreciates at a flat rate of 17.5% per annum. If its scrap value is $875, find:

**a** the book value of the machine after two years
**b** after how many years the machine will be written off

**9** A stereo system purchased for $1200 incurs 12% per annum reducing balance depreciation.

**a** Find the book value after 7 years.
**b** What is the total depreciation after 7 years?
**c** If the stereo has a scrap value of $215, in which year will this value be reached?

**10** A car costing $38 500 depreciates at a rate of 9.5% per year. Give your answers to the following to the nearest dollar.

**a** What is the book value of the car at the end of five years?
**b** What is the total amount of depreciation after five years?
**c** If the car has a scrap value of $10 000, in which year will this value be reached?

**11** A machine has a book value after 10 years of $13 770. If it has depreciated at a reducing balance rate of 8.2% per annum, what was the initial cost of the machine?

**12** After depreciating at a reducing balance rate of 12.5% per annum, a yacht is now worth $56 100. What was the yacht worth when it was new six years ago to the nearest $100?

**13** What reducing balance rate would cause the value of a car to drop from $8000 to $6645 in three years?

## 21.6 Applications of Finance Solvers

There are many more complex problems in business mathematics which are best solved with technology. Your calculator has an application called *Finance Solver* (*TI-nspire*) or *Finance* (*ClassPad*), which allow you to investigate three more complex situations:

- reducing balance loans
- annuities
- adding to the investments

Each of these is an application of the following formula (sometimes called the annuities formula).

**The annuities formula**

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

Here:

- $\$A$ is the amount owing or invested after $n$ years
- $\$P$ is the amount borrowed or invested
- $R = 1 + \frac{r}{100}$ where $r$% per annum, compounded annually, is the amount of interest on the loan or investment
- $\$Q$ is the amount paid each year

Any calculations based on this formula can be carried out readily using the finance solver application.

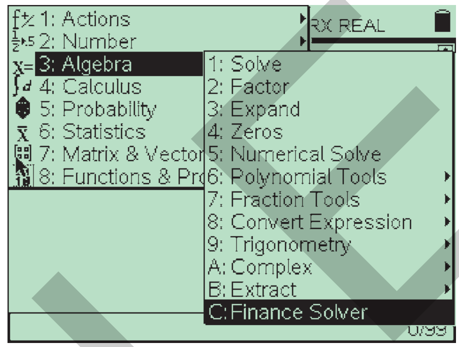
- **N** is the total number of payments
- **I(%)** is the annual interest rate
- **PV** is the present value of the loan or investment
- **Pmt (**or **PMT)** is the amount paid at each payment
- **FV** is the future value of the loan or investment
- **PpY (**or **PY)** is the number of payments made per year
- **CpY (**or **CY)** is the number of times per year the interest is compounded (and is almost always the same number of payments made per year)
- **PmtAt** (on the TI-Nspire CAS) is concerned with whether the interest is compounded at the end or at the beginning of the time period; leave it set at **END**. This is the assumed setting on the ClassPad.

## How to use Finance Solver on the TI-Nspire CAS

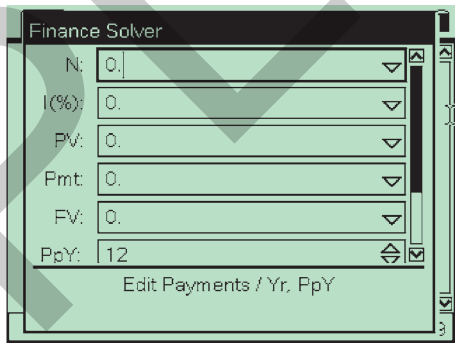
### Steps

1 Start a new document by pressing (ctrl) + (N), and select **1:Add Calculator**.

2 Press (menu)/**3:Algebra/C:Finance Solver**.




3 To use **Finance Solver** you need to know the meaning of each of its symbols. These are as follows:




- **N** is the total number of payments
- **I(%)** is the annual interest rate
- **PV** is the present value of the loan or investment
- **Pmt** is the amount paid at each payment
- **FV** is the future value of the loan or investment
- **PpY** is the number of payments per year
- **CpY** is the number of times the interest is compounded per year. (It is almost always the same as **PpY**.)
- **PmtAt** is used to indicate whether the interest is compounded at the end or at the beginning of the time period. Leave this set at **END**.

**Note:** Use (tab) to move between boxes and use ▼ to make a selection within a box. Press (tab) to move down to the next entry box. Press (CAP) + (tab) to move up.

4 When using **Finance Solver** to solve loan and investment problems, there will be one unknown quantity. To find its value, move the cursor to its entry box and press (enter) to solve.

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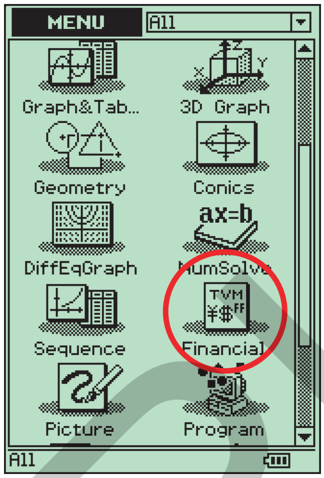
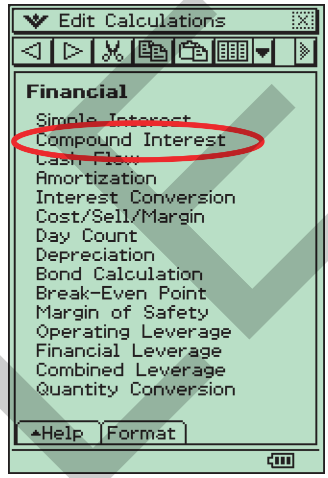
## How to use the Financial application on the ClassPad

### Steps

1 From the application menu, locate and open the **Financial** application.

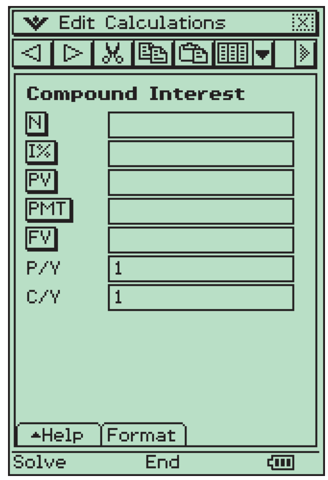



2 The **Financial** screen gives a variety of business mathematics applications. The one that is needed is **Compound Interest**. Tapping this option will open a related dialog box, as shown below.

3 To use the **Compound Interest** application, you need to know the meaning of each of its symbols. These are as follows:

- **N** is the total number of payments
- **I%** is the annual interest rate
- **PV** is the present value of the loan or investment
- **PMT** is the amount paid at each payment
- **FV** is the future value of the loan or investment
- **P/Y** is the number of payments per year
- **C/Y** is the number of times interest is compounded per year. (It is almost always the same as **P/Y**.)




**Note:** The **Help** tab at the bottom of the screen also gives these definitions.

4 When using the **Compound Interest** application to solve loan and investment problems, there will be one unknown quantity. To find its value tap its box.

Now we can consider each of applications of the finance solver separately.

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## Reducing balance loans

Reducing balance loans were introduced in Section 20.4. Essentially, this describes a situation where a loan is taken out under compound interest, and period repayments are made. Often, we are interested in determining the repayment so that both the loan and the interest are repaid over a specified period of time. This is illustrated in the following example.

**Example 17** **Determining the amount of the repayment for a reducing balance loan**

Simone borrows $10 000 at 8% per annum compounded monthly. She intends to make monthly repayments, and wants to pay off the loan in 5 years. How much should she repay each month?

### Solution

**1** Open **Finance Solver** on your calculator and enter the information below, as shown opposite.

- **N**: 60 (number of monthly payments in 5 years)
- **I%**: 8 (annual interest rate)
- **PV**: 10 000 (positive as this is the amount the bank has given to you)
- **Pmt** or **PMT** (the unknown payment): leave blank or **Clear**
- **FV**: 0 (loan is to be paid off completely)
- **Pp/Y**: 12 (monthly payments)
- **Cp/Y**: 12 (interest compounds monthly)

| | |
|---|---|
| N: | 60 |
| I%: | 8 |
| PV: | 10000 |
| Pmt or PMT: | |
| FV: | 0 |
| Pp/y or P/Y: | 12 |
| Cp/y or C/Y: | 12 |

**2** Solve for the unknown. On the **TI-Nspire**: Move the cursor to the **Pmt or PMT** entry box and press (enter) to solve.
**ClassPad**: Tap on the **PMT** box to the left.
The amount –202.7639 . . . now appears in the **Pmt or PMT** entry box.
**Note:** The sign of the payment is negative because it is the amount that Simone must give back (repay) to the bank each month.

| | |
|---|---|
| N: | 60 |
| I%: | 8 |
| PV: | 10000 |
| Pmt or PMT: | –202.7639… |
| FV: | 0 |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

Simone must repay the bank $202.76 per month.

**3** Write your answer.

Using a finance solver we can solve for any of the variables listed.

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## Example 18 Determining the amount owed and the number of repayments for a reducing balance loan

Andrew borrows $20 000 at 7.25% per annum compounded monthly, and makes monthly repayments of $200.

**a** How much does he owe after three years?

**b** How long will it take him to pay out the loan? Give your answer to the nearest month.

### Solution

**a** **1** Open the **Finance Solver** on your calculator and enter the information below, as shown opposite.

- **N**: 36 (number of monthly payments in 3 years)
- **I%**: 7.25 (annual interest rate)
- **PV**: 20 000 (positive, as this is the amount the bank has given to you)
- **Pmt** or **PMT**: –200
- **FV** (the unknown quantity): leave blank or **Clear**
- **Pp/Y**: 12 (monthly payments per year)
- **Cp/Y**: 12 (interest compounds monthly)

| | |
|---|---|
| N: | 36 |
| I%: | 7.25 |
| PV: | 20000 |
| Pmt or PMT: | –200 |
| FV: | |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

**2** Solve for the unknown. On the

**TI-Nspire**: Move the cursor to the **FV** entry box and press (enter) to solve.

**ClassPad**: Tap on the **FV** box on the left.

The amount –16826.97 . . . now appears in the **FV** entry box.

**Note:** The sign of the future value is negative, indicating that this is money that is still owed to the bank and must, eventually, be paid back.

| | |
|---|---|
| N: | 36 |
| I%: | 7.25 |
| PV: | 20000 |
| Pmt or PMT: | –200 |
| FV: | –16826.97… |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

**3** Write your answer.

*After 3 years, Andrew still owes the bank $16,826.97*

**b** **1** **N**, the total number of payments, is now the unknown. Leave blank or **Clear**.

Change **FV** to **0** to indicate the loan is to be fully paid out.

All other values stay the same.

| | |
|---|---|
| N: | |
| I%: | 7.25 |
| PV: | 20000 |
| Pmt or PMT: | –200 |
| FV: | 0 |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

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2 Solve for the unknown. On the **TI-Nspire**: Move the cursor to the **N** entry box and press (enter) to solve.
**ClassPad**: Tap on the **N** box on the left.
The amount 153.8580 . . . now appears in the **N** entry box.

3 Write your answer, correct to the nearest month.

| | |
|---|---|
| N: | 153.8580… |
| I%: | 7.25 |
| PV: | 20000 |
| Pmt or PMT: | –200 |
| FV: | 0 |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

It will take Andrew 154 months to fully pay out the loan.

## Annuities

An **annuity** is the opposite of a reducing balance loan. Here, an amount of money is invested at a fixed rate of compound interest, and then periodical withdrawals are made until there is no money left. Once again, any of the variables can be solved for using a finance solver, but we are generally interested in the amount of the payment which can be made, or the length of time for the annuity will last.

### Example 19 Determining the amount of the payment and the number of payments for an annuity

Joe purchases a $200 000 annuity which is invested at 5% compound interest per annum compounded monthly.

**a** If he wishes to be paid monthly payments for 10 years, how much will he receive each month?

**b** If he receives a regular monthly payment of $3000, how long will the annuity last? Give your answer to the nearest month.

### Solution

**a** 1 Open the **Finance Solver** on your calculator, as shown. **PV** is negative as this is the amount Joe gives the bank to purchase the annuity.

2 Solve for **Pmt**.
**Note:** The sign of the **Pmt** is positive because it is the payment that Joe receives from the bank each month.

3 Write your answer.

| | |
|---|---|
| N: | 120 |
| I%: | 5 |
| PV: | –200000 |
| Pmt or PMT: | 2121.31… |
| FV: | 0 |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

Joe receives $2121.31 each month.

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**b** **1** Change the payment **PmT** to 3000 (positive, as this is money that the bank pays to Joe) and solve for **N**.

| | |
|---|---|
| N: | 78.263 ... |
| I%: | 5 |
| PV: | –200000 |
| Pmt or PMT: | 3000 |
| FV: | 0 |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

**2** Write your answer, correct to the nearest month.

Joe's annuity will last 78 months.

## Adding to an investment

Suppose that an amount of money is invested in an account which pays compound interest, and that regular payments are also made into the account. This is also called an **annuity investment**. The amount in the investment at any time can be determined using TVM Solver, as can the other variables in the equation.

### Example 20 Determining the amount of an investment that is added to on a regular basis

Larry invests $500 000 at 5.5% per annum compounded monthly. He makes regular deposits of $500 per month into the account. What is the amount of the investment after five years?

**Solution**

**1** Set up your **Finance Solver** as shown. Note that **PV** and **Pmt or PMT** are negative as these amounts are put into the account. Solve for **FV**.

**Note:** **FV** will be positive as this is the amount that will be given back to Larry at the end of the investment period.

| | |
|---|---|
| N: | 60 |
| I%: | 5.5 |
| PV: | –500000 |
| Pmt or PMT: | –500 |
| FV: | 692292.297.. |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

**2** Write your answer.

After 5 years, Larry's investment will be worth $692,292.30.

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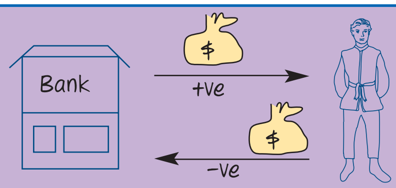
As you can see from the examples in this section, finance solver is an extremely useful application. However, you need to be very careful of the sign when entering or interpreting each of the variables. The signs convention used in this section is summarised in the following table:

| | | |
|---|---|---|
|  | | **Rules**<br>Bank gives you money: positive (+ve)<br>You give Bank money: negative (-ve) |
| **Loan** | PV positive<br>PMT negative<br><br>FV 0 or negative | Bank gives (lends) you money<br>You pay off the loan by making regular payments to the bank.<br>At the end of the loan, it is either fully paid out (FV = 0) or you still have to pay money to the bank. |
| **Annuity** | PV negative<br>PMT positive<br><br>FV 0 or positive | You give money to the bank.<br>The bank makes regular payments to you from the investment.<br>At the end of the annuity, there is either no money left (FV = 0) or the bank pays you back what is left. |
| **Investment which is added to on a regular basis** | PV negative<br>PMT negative<br><br>FV positive | You give money to the bank.<br>You increase the amount you have invested by making regular payments to the bank.<br>At the end of the investment the bank pays you the total final value of your investment. |

Sometime in the future you will most likely want to borrow money, and there are many possible lenders who will offer a variety of terms and conditions for the loan. It is therefore important that you are able to compare the cost of these loans, in order to be able to make an informed decision about which is the best one.

**Example 21** **Comparing loans**

Karim intends to borrow \$10 000 to buy a car, which he hopes to pay off over 5 years. He has the choice of the following plans:

Plan A: A flat rate of interest of 7.5%

Plan B: Interest at 10% per annum, reducing quarterly

Determine the total repayments that will be made on each loan, and hence which plan Karim should choose.

### Solution

1 Calculate the interest paid under Plan A using the simple interest formula with $P = 10\,000$, $r = 7.5$ and $t = 5$.

PlanA

$$\text{Interest paid} = \frac{Prt}{100}$$

$$= \frac{7.5}{100} \times 10\,000 \times 5 = \$3750$$

PlanB

2 Calculate the interest paid with plan B. Start by using your **Finance Solver** to determine the payments **Pmt** with

**N:** 20
**I%:** 10
**PV:** 10 000 (positive, as this is the amount the bank has given to or lent Karim)
**FV:** 0
**Pp/Y:** 4
**Cp/Y:** 4 (quarterly payments)

**Note:** **Pmt or PMT** will be negative as this amount will need to be paid back to the lender at the end of each quarter.

| | |
|---|---|
| N: | 20 |
| I%: | 10 |
| PV: | 10000 |
| Pmt or PMT: | –641.47… |
| FV: | 0 |
| Pp/y or P/Y: | 4 |
| Cp/Y or C/Y: | 4 |

3 Determine the total amount paid with 20 payments of \$641.47 each.

$$\text{Total paid} = 20 \times 641.47 = 12\,829.40$$

4 Determine the interest paid under Plan B.

$$\text{Interest paid} = 12\,829.40 - 10\,000 = 2829.40$$

5 Compare the interest paid under each plan and choose the one where the lesser interest is paid.

Karim is $3750 - 2829.40 = \$920.60$ better off under Plan B.

## Interest only loans

On occasion people choose to enter into arrangements with their banks to pay only the interest on the loan, and none of the principal. These are known as **interest only loans** and are useful when the item to be purchased under the loan will increase in value anyway (such as land perhaps) and will eventually be sold to pay out the loan at the end of the time period.

## Example 22 Interest only loans

Jane borrows $500 000 to buy shares. If the interest on the loan is 6.65% per annum, compounding monthly, what will be her monthly repayment on an interest only loan?

### Solution

1 We will consider the situation for 1 year only; all other years will be the same. Using your **Finance Solver**, solve for **Pmt** with

**N:** 12
**I%:** 6.65
**PV:** 500 000
**FV:** −500 000 (negative, as Jane will eventually have to pay this money back to the lender)
**Pp/Y:** 12
**Cp/Y:** 12

**Note:** **Pmt or PMT** will be negative as this amount will need to be paid back to the lender at the end of each month.

| | |
|---|---|
| N: | 12 |
| I%: | 6.65 |
| PV: | 500000 |
| Pmt or PMT: | –2770.83… |
| FV: | –500000 |
| Pp/y or P/Y: | 12 |
| Cp/Y or C/Y: | 12 |

2 Write your answer.

Jane's monthly repayments are $2770.83.

The repayment on the interest only loan is equivalent to paying only the simple interest due on the principal for one year. This can be readily verified using the information in Example 22.

## Perpetuities

A perpetuity is an investment which pays out an equal amount, hopefully forever! For example, you might want to start a scholarship at your school where, every year, a student will receive $1000. You want this scholarship to continue indefinitely, even after you are long gone. The question is, how much money will it cost you? Consider the formula for the amount of an annuity:

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

In the case of perpetuity, $A$ is equal to $P$. Substituting this value, rearranging and simplifying we find:

In a perpetuity, if an amount $\$P$ is invested at an interest rate of $r$% per annum, and a regular payment of $\$Q$ per annum is made, then:

$$P = \frac{100Q}{r} \qquad \text{or} \qquad Q = \frac{Pr}{100}$$

Since in a perpetuity the interest is withdrawn each time it is paid it can also be considered as an application of simple interest. This can be seen from the formula for $Q$ above.



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TI-Nspire & Casio ClassPad material in collaboration with Brown and McMenamin

**Example 23** **Perpetuity investment amount**

Suppose Richard wishes to start a scholarship, where every year a student receives $1000. If the interest on the initial investment averages 3% per annum, how much should be invested?

**Solution**

Substitute $Q = \$1000$ and $r = 3$ into the formula for a perpetuity.

$$P = \frac{100Q}{r} = \frac{100 \times 1000}{3} = 33\,333.33$$

Answer: Invest $33 333.33

**Example 24** **Perpetuity payment**

Elizabeth places her superannuation payout of $500 000 in a perpetuity which will provide a monthly income without using any of the principal. If the interest rate on the perpetuity is 6% per annum compounding monthly, what monthly payment will Elizabeth receive?

**Solution**

1 To determine how much Elizabeth will receive per annum substitute $P = \$500\,000$ and $r = 6$ into the formula to find the payment, $Q$

2 Since this is the annual payment we divide by 12 to find the monthly payment.

$$Q = \frac{Pr}{100} = \frac{500000 \times 6}{100}$$
$$= \$30\,000$$

$$\text{Monthly payment} = \frac{30\,000}{12} = \$2500$$

## Exercise 21F

1 A reducing balance loan of $90 000 is borrowed at the monthly adjusted interest rate of 11% per annum. Monthly instalments of $857 are to be paid over a period of 30 years. Find:

- **a** the amount still owing after 10 years
- **b** the amount still owing after 20 years
- **c** the total amount paid back on the loan after the full period 30 years
- **d** the amount of interest paid on the loan

2 A building society offers $120 000 home loans at the compound interest rate of 10.25% per annum adjusted monthly.

- **a** If repayments are $1100 per month, calculate the amount still owing on the loan after 12 years.
- **b** If the loan is to be fully repaid after 12 years, calculate the amount of the monthly repayment.

3 Interest on a reducing balance loan of $65 000 is compounded quarterly at an interest rate of 12.75% per annum. Calculate the quarterly repayment if:

- **a** the amount still owing after 10 years is $25 000

b the amount still owing after 20 years is \$25 000
c the loan is fully repaid after 10 years
d the loan is fully repaid after 20 years

4 A couple negotiates a 25-year mortgage of \$150 000 at a fixed rate of 7.5% per annum compounded monthly for the first 7 years, then at the market rate for the remainder of the loan. They agree to monthly repayments of \$1100 for the first 7 years. Calculate:
a the amount still owing after the first 7 years
b the new monthly repayments required to pay off the loan if after 7 years the market rate has risen to 8.5% per annum

5 Dan arranges to make repayments of \$450 per month to repay a loan of \$20 000 with interest being charged at 9.5% per annum compounded monthly. Find:
a the number of monthly repayments required to pay out the loan (to the nearest month)
b the amount of interest charged

6 Joan considers taking out a loan on the terms given in Question 5. However, she decides that she can afford higher monthly repayments of \$550.
a How long does it take her to pay off her loan (to the nearest month)?
b What is her saving in interest by paying the higher monthly instalment?

7 If, after one year, the interest rate in Question 6 is reduced to 7.5% per annum, and Joan continues to pay her \$550 monthly repayments, how long will it take her to pay off her loan? How much less interest does she pay at this lower interest rate?

8 Stephanie purchases a \$40 000 annuity with interest paid at 7.5% per annum compounded monthly. If she wishes to receive a monthly payment for 10 years, how much will she receive each month?

9 Lee purchases an annuity for \$140 000, with interest of 6.25% per annum compounded monthly. If he receives payments of \$975 per month, how long will the annuity last? Give your answer to the nearest month.

10 Raj purchases an annuity for \$85 500, with interest of 7.25% per annum compounded quarterly.
a If he receives quarterly payments for 10 years, how much will he receive each quarter?
b If he receives a regular quarterly payment of \$5000, how long will the annuity last? Give you answer to the nearest quarter.

11 Bree has \$25 000 in an account which pays interest at a rate of 6.15% per annum compounding monthly.
a If she makes monthly deposits of \$120 to the account, how much will she have in the account at the end of 5 years?
b If she makes monthly withdrawals of \$120 from the account, how much will she have in the account at the end of 5 years?

**12** Jarrod saves \$500 per month in an account which pays interest at a rate of 6% per annum compounding monthly.

**a** If he makes monthly deposits of \$500 to the account, how much will he have in the account at the end of 10 years?

**b** Suppose that after 10 years of making deposits, Jarrod starts withdrawing \$500 each month from the account. How much will he have in the account at the end of another 10 years?

**13** Mr and Mrs Kostas decide to borrow \$25 000 to help them finance the construction of their swimming pool. They consider two loan repayment options:

Loan option A: Monthly repayments of 7.5% per annum compounded monthly

Loan option B: Quarterly repayments at 7.5% per annum compounded quarterly

They wish to pay off the loan over 5 years. Calculate to the nearest dollar for each loan:

**a** the total repayment

**b** the total interest paid

and hence decide which, if either, is the better loan.

**14** If Mr and Mrs Kostas of Question 11 choose Loan option A, how much interest do they pay if the interest rate is increased by 0.5%?

**15** An amount of \$35 000 is borrowed in 2005 for 20 years at 10.5% per annum compounded monthly.

**a** What are the repayments for the loan?

**b** How much interest is paid on the loan over the 20-year period?

**c** How much is still owing at the end of 4 years?

After four years, the interest rate rises to 13.75% per annum.

**d** What are the new repayments that will see the amount repaid in a total of 20 years?

**e** How much extra must now be repaid on the loan over the term of 20 years?

**16** A couple put a \$20 000 down-payment on a new home and arranged to pay off the rest in monthly instalments of \$625 for 30 years at a monthly compounded interest rate of 8.5% per annum.

**a** What was the selling price of the house to the nearest cent?

**b** How much interest will they pay over the term of the loan?

**c** How much do they owe after 6 years?

After 6 years the interest rates climb by 0.9%. The couple must now extend the period of their loan in order to pay it back in full.

**d** How much do they still owe after the original 30-year period?

**e** Will they ever repay the loan at their original monthly repayment of \$625?

**f** Calculate the new monthly repayment amount required if the couple still wishes to pay off the loan in 30 years.

**17** A credit institution offers loans of up to \$12 000 at an interest rate of $9\frac{1}{3}$% per annum calculated on the principal.

**a** Calculate the interest charged on a loan of \$9500 after three years.
**b** Calculate the monthly repayments on the loan in **a** if it is to be fully repaid in five years.
**c** If monthly repayments of \$370.75 are made, how long does it take to fully repay the loan?

**18** A rival credit institution to that in Question 17 offers loans of up to \$12 000 but with an interest rate of $9\frac{1}{3}$% per annum calculated on the outstanding balance before monthly repayments are made.

**a** Calculate the amount saved in repayments per month with this credit institution if the loan of \$9500 is to be fully repaid over five years.
**b** Calculate the amount saved in total, to the nearest dollar.

**19** A flat rate loan over 6 years at 12.75% per annum amounted to a repayment of \$12 500.

**a** How much was originally borrowed?
**b** Calculate the quarterly repayments.
**c** Compare the savings of a reducing balance loan by working out the quarterly repayments with interest set at 12.75% per annum compounded quarterly.
**d** How much is saved over the full 6-year period by adopting a reducing balance loan?

**20** A personal loan of \$7500 is taken out at 11.5% per annum over 4 years.

**a** Calculate the total amount to be repaid:
  **i** if the loan was a flat rate loan
  **ii** if the loan was a reducing balance loan with monthly repayments
**b** What flat rate payment of interest would the monthly repayment in **a** part **ii** be equivalent to?

**21** Georgia borrows \$100 000 to buy an investment property. If the interest on the loan is 7.15% per annum, compounding monthly, what will be her monthly repayment on an interest only loan?

**22** In order to invest in the stockmarket, Jamie takes out an interest only loan of \$50 000. If the interest on the loan is 8.15% per annum, compounding monthly, what will be his monthly repayments?

**23** Jackson takes out an interest only loan of \$30 000 from the bank to buy a painting, which he hopes to resell at a profit in 12 months' time. The interest on the loan is 9.25% per annum, compounding monthly, and he makes monthly payments on the loan. How much will he need to sell the painting for in order not to lose money?

**24** Geoff wishes to set up a fund so that every year \$2500 is donated to the RSPCA in his name. If the interest on his initial investment averages 2.5% per annum, compounded annually, how much should he invest?

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**25** Barbara wishes to start a scholarship which will reward the top mathematics student each year with a $500 prize. If the interest on the initial investment averages 2.7% per annum, compounded annually, how much should be invested? Give your answer to the nearest dollar.

**26** Cathy wishes to maintain an ongoing donation of $5500 per year to the Collingwood Football Club. If the interest on the initial investment averages 2.75% per annum, compounded annually, how much should she invest?

**27** Craig wins $1 000 000 in a lottery and decides to place it in a perpetuity which pays 5.75% per annum interest, compounding monthly. What monthly payment does he receive?

**28** Suzie invests her inheritance of $642 000 in a perpetuity which pays 6.1% per annum compounding quarterly. What quarterly payment does she receive?



MC TEST D&D TEST

Review

## Key ideas and chapter summary

| | |
|---|---|
| **Income tax** | Money deducted by your employer and sent to the government to pay for facilities such as hospitals, schools, roads is called **income tax**. |
| **Tax subdivisions** | A classification of income levels where different tax rates are applied. Also called a tax bracket. |
| **Marginal rate** | The percentage applied to your income to calculate the tax applicable to a particular tax subdivision is called the **marginal rate**. |
| **Gross salary** | Your **gross salary** is the amount you are paid before any deductions are made. |
| **Net salary** | The amount you actually receive is called your **net salary**. |
| **Capital gains tax** | Capital gains tax is the tax paid on the profit made from an investment. |
| **GST** | The **goods and services tax**, applicable on most items except fresh food, is charged by the government at a rate of 10%. |
| **Stamp duty** | Stamp duty is the duty or tax charged by the government on various commercial transactions. |
| **Minimum monthly balance** | The lowest amount the account contains in each calendar month is called the **minimum monthly balance**. |
| **Minimum daily balance** | The lowest amount the account contains on a particular day is called the **minimum daily balance**. |
| **Hire purchase** | **Hire purchase** is where the purchaser hires an item from the vendor and makes periodical payments at an agreed rate of interest. At the end of the period of the agreement, the item is owned by the purchaser. |
| **Flat rate of interest** | The interest paid given as a percentage of the original amount owed, given by: **flat interest rate per annum**, $r_f = \frac{100I}{Pt}$ Where $I$ = total interest paid, $P$ = principal owing after the deposit has been deducted, and $t$ = the number of years. |

**Effective interest rate** — The interest paid given as a percentage of the average amount owed, given by:

$$\textbf{effective interest rate per annum}, r_e = \frac{100I}{Pt} \times \frac{2n}{(n+1)}$$

Where $I$ = total interest paid, $P$ = principal owing after the deposit has been deducted, $t$ = number of years and $n$ = number of payments made in total.

**Inflation** — **Inflation** is the upward movement in the general level of prices.

**Purchasing power** — **Purchasing power** describes what you can actually buy with your money.

**Depreciation** — **Depreciation** is the amount by which the value of an item decreases over time.

**Book value** — The depreciated value of an item is called its **book value**.

**Scrap value** — The value at which an item is no longer of value to the business, so it is replaced, is called its **scrap value**.

**Unit cost depreciation** — The process of depreciating an item on the basis of the amount of use, that is, according to the number of units it has produced is called **unit cost depreciation**.

**Unit cost** — The cost per unit for production is determined by:

$$\textbf{unit cost} = \frac{\text{purchase price} - \text{scrap value}}{\text{total production}}$$

**Flat rate depreciation** — **Flat rate depreciation** is when the value of an item is reduced by the same percentage of the purchase price, or amount, for each year it is in use. It is equivalent but opposite to simple interest. The depreciation of an item ($\$D$), which has a purchase price of $\$P$, and which is depreciating at a flat rate of $r$% per annum annually for a time period of $t$ years is:

$$D = \frac{Prt}{100}$$

**Book value (flat rate depreciation)** — The **book value** of an item ($\$V$), which has a purchase price of $\$P$, and which is depreciating at a flat rate of $r$% per annum annually for a time period of $t$ years is:

$$V = P - D = P - \frac{Prt}{100}$$

**Reducing balance depreciation** — **Reducing balance depreciation** is when the value of the item is reduced by a constant percentage for each year it is in use.

It is the equivalent, but opposite, situation to compound interest, and the amount of the deprecation is given by:

$$D = P - V$$

where $D$ is depreciation, $P$ is the purchase price and $V$ is the book value.

**Book value (reducing balance depreciation)**

The **book value** of an item ($\$V$), which has a purchase price of $\$P$, and which is depreciating at a rate of $r\%$ per annum compounded annually for a time period of $t$ years is:

$$V = P \times \left(1 - \frac{r}{100}\right)^t$$

**TVM Solver**

TVM solver is a facility of the graphics calculator where the annuities formula is solved. We can use it to solve problems involving reducing balance loans, annuities and investments where regular payments are made.

**Interest only loans**

An **interest only loan** is a loan where only the interest in paid on the loan, and none of the principal. In terms of the annuities formula, that is a loan where $A = P$.

**Perpetuity**

A perpetuity is an investment where an equal amount is paid out forever. If an amount $\$P$ is invested at an interest rate of $r\%$ per annum, and a regular payment of $\$Q$ per annum is paid, then:

$$P = \frac{100Q}{r} \quad \text{and} \quad Q = \frac{Pr}{100}$$

## Skills check

Having completed this chapter you should be able to:

- use the table of tax subdivisions and marginal rates to determine the income tax payable on any salary
- use the table of tax subdivisions and marginal rates to determine capital gains tax payable on the profit on an investment
- calculate the amount of GST that should be added to an account
- given the total value of an account after GST has been added, determine the price without GST
- given the purchase price of a property, determine the stamp duty payable
- determine the interest payable on a bank account, whether paid on the minimum monthly balance or the minimum daily balance, over a period when several transactions have been made

- determine the flat rate of interest per annum for a hire purchase agreement
- determine the effective interest rate per annum for a hire purchase agreement
- determine the effective interest rate per annum from the flat interest rate per annum
- determine the effect of inflation on prices (using Equation Solver where appropriate)
- determine the effect of inflation on purchasing power (using Equation Solver)
- calculate unit cost depreciation, book value and the length of time an asset will be in use under unit cost depreciation
- calculate flat rate depreciation, book value and the length of time an asset will be in use under flat rate depreciation
- calculate reducing balance depreciation, book value and the length of time an asset will be in use under reducing balance rate depreciation (using Equation Solver)
- determine the value of any of $A$, $P$, $r$, $Q$, or $n$ from the annuities formula using TVM Solver (including the case of interest only loans)
- determine the amount $P$ which should be invested at a given interest rate $r$ to ensure an annual perpetuity of $Q$

## Multiple-choice questions

1 Ben is paid a salary is $37 950 per year. How much does he pay each year in income tax, correct to the nearest dollar?

**A** $7557 **B** $2652 **C** $5431 **D** $9585 **E** $5432

2 In one financial year Lizzie makes a profit of $34 346 buying and selling antiques. If her other income for that financial year is $5000, how much capital gains tax will she pay on the profit she makes on the antiques, correct to the nearest dollar?

**A** $4676 **B** $10 304 **C** $5839 **D** $28 169 **E** $7976

3 If a mobile phone is advertised for sale for $399, including GST, how much would the phone have cost without GST?

**A** $39.90 **B** $438.90 **C** $362.73 **D** $359.10 **E** $389.00

4 Phillip buys a house for $452 500. What is the value of the stamp duty payable on this purchase?

**A** $20 250 **B** $27 150 **C** $22 810 **D** $24 887.50 **E** $10 660

5 Interest in Terry's bank account is calculated on the minimum monthly balance. The interest rate is 6% per annum. The complete statement for the month of July is shown. Assuming no other deposits or further withdrawals were in July, the interest for July is closest to:

| *Date* | *Credit* | *Debit* | *Balance* |
|---|---|---|---|
| 1 July | | | $ 2137.00 |
| 13 July | | $ 1025.00 | $ 1112.00 |

**A** $5.56 **B** $66.72 **C** $10.69 **D** $128.22 **E** $7.69

6 Tim borrows $10 000 at a flat rate of 11% per annum to be repaid in 60 equal monthly repayments. Which one of the following is true?

**A** The effective interest rate equals 11% as it is the agreed interest rate put into effect upon signing the contract.
**B** The effective interest rate is less than 11% as the amount of interest owing decreases with each payment.
**C** The effective interest rate is greater than 11% as the amount owing is reduced at each repayment.
**D** Interest for each payment is calculated at 11% per annum on the remaining balance owing each month.
**E** Interest for each payment for the year is calculated at 11% per annum on the remaining balance owing at the start of each year.

7 A television set can be bought for $995 or $100 deposit and $20 per week for one year. The flat rate of interest per annum this represents is closest to:

**A** 14.6% **B** 16.2% **C** 11.6% **D** 25.0% **E** 4.5%

8 Repayments on Frank's hire-purchase agreement are based on a flat interest rate per annum of 11%, and he is to make 50 repayments. This represents an effective interest rate per annum of:

**A** 22% **B** 55% **C** 5.5% **D** 23% **E** 21.6%

9 Suppose that the price of a newspaper is $1.20 today. What will be the price of the newspaper in 20 years' time if the average annual inflation rate is 2.5%?

**A** $1.23 **B** $1.54 **C** $2.02 **D** $1.97 **E** $1.80

10 Cash savings of $62 000 are put into a safety deposit box in the bank in 2006. The purchasing power of this amount in 15 years' time if the average inflation rate over this period is 4.1% is closest to:

**A** $50 255 **B** $38 130 **C** $113 280 **D** $33 934 **E** $38 813

11 A new computer costs $3400. If depreciation is calculated at 15% per annum (reducing balance), then the computer's reduced value at the end of 4 years, in dollars, will be closest to:

**A** $2040 **B** $2890 **C** $1944 **D** $1360 **E** $1775

12 A loan of $6000, plus interest, is to be repaid in full in 12 quarterly instalments. Interest at 10% per annum is calculated on the remaining balance each quarter. The amount of the repayment required to pay out the loan is closest to:

**A** $585 **B** $550 **C** $650 **D** $527 **E** $500

13 Monthly withdrawals of $220 are made from an account which has an opening balance of $35 300, invested at 7% per annum compounding monthly. The balance of the account after one year is closest to:

**A** $35 125 **B** $40 578 **C** $32 660 **D** $33 500 **E** $35 211

Review

**14** Nathan makes regular monthly deposits of $430 into his bank account, which pays 6.1% per annum compounding quarterly. If his opening balance is $5000, the balance in the account after 2 years is closest to:

**A** $5300 **B** $15 320 **C** $16 600 **D** $15 500 **E** $16 000

**15** Paula borrows $12 000 from a bank, to be repaid over 5 years. Interest of 12% per annum is charged monthly on the amount of money owed. If Paula makes monthly repayments, then the amount she owes at the end of the second year is closest to:

**A** $9120 **B** $6410 **C** $8040 **D** $5590 **E** $2880

**16** Richard takes out an interest only loan of $25 000 for 5 years. The interest on the loan is 7.65% per annum, compounding monthly. The monthly repayment on the loan is closest to:

**A** $2171 **B** $159 **C** $187 **D** $1913 **E** $383

**17** Leon wishes to set up a fund so that every year $3000 is donated in his name to a promising athlete. If the interest on his initial investment averages 3.5% per annum, compounded annually, the amount he should invest is closest to:

**A** $90 000 **B** $85 700 **C** $33 500 **D** $120 000 **E** $42 300

## Extended-response questions

**1** The Johnstones bought a new car priced at $24 800 and paid a deposit of $5000 cash. They borrowed the balance of the purchase price at simple interest. They then agreed to repay the loan, plus interest, in equal monthly payments of $550 over 4 years.

**a** Calculate the total amount of interest to be repaid over the term of this loan.

**b** Calculate, to one decimal place, the annual simple interest rate charged on this loan.

**c** The value of this car was depreciated using a reducing balance method at a rate of 15% per year.

**i** Calculate, to the nearest dollar, the depreciated value of the car after 3 years.

**ii** Calculate, to two decimal places, the annual percentage rate of depreciation that would reduce the value of the car from $24 800 to $18 000 in 3 years.

**2** The Andersons were offered a $24 800 loan to pay the total cost of a similar new car. Their loan is to be repaid in equal monthly repayments of $750, except for the last month when less than this will be required to pay out the loan. They will pay 10.8% interest per annum calculated monthly on the reducing balance.

**a** Calculate the least number of months needed to repay this loan plus interest.

**b** Calculate, to the nearest cent, the amount of the final repayment.

**c** When the Andersons took out their loan they had the choice of making monthly repayments of \$750 or quarterly payments of \$2250. They chose to make monthly repayments of \$750. In either case they would have to pay 10.8% interest per annum calculated monthly on the reducing balance.
In terms of the total amount of money they would have to pay to repay the loan, did they make the correct decision? Explain your answer without making any further calculations.

**3** When a family bought their home they borrowed \$100 000 at 9.6% per annum compounded quarterly. The loan was to be repaid over 25 years in equal quarterly repayments.

**a** How much of the first quarterly repayment went towards paying off the principal?

**b** The family inherits some money and decides to terminate the loan after 10 years and pay what is owing in a lump sum. How much will this lump sum be?

**4** Kelly is saving money to put towards the purchase of a new car. She has had her present car for five years and it cost her \$22 500. To know how much money she will need to borrow, she must estimate the current value of her present car.

**a** What will be her estimate of the current value of her car if she assumes that it has depreciated at a flat rate of 12% per annum?

**b** What will be her estimate of the current value of her car if she assumes that it has depreciated at a rate of 16% per annum on its reducing value? Give your answer to the nearest one hundred dollars.

**5** Kelly trades her old car in for \$10 500 on a car worth \$35 400. To repay what she still owes, Kelly considers two options:

Option 1: borrowing all the money owed
Option 2: using part of her savings to reduce the amount she would have to borrow

**a** Following Option 1, Kelly can obtain a reducing balance loan at an interest rate of 7.55% per annum adjusted monthly to be paid off in monthly repayments over three years.

**i** How much would she repay in total? Give your answer to the nearest ten dollars.

**ii** How much of this amount is interest?

**b** Following Option 2, Kelly could withdraw \$3000 from her savings account and use it with her trade-in to reduce the amount of money she would have to borrow. If she takes out this loan on the same terms as in part **a**:

**i** How much would she now have to borrow?

**ii** How much would she repay in total? Give your answer to the nearest ten dollars.

**iii** How much of this amount is interest?

*(cont'd.)*

**c** Kelly wants to compare the two options for financing her car as set out in part **b**.

**i** Calculate the interest saved over the 3-year period if Option 2 is adopted. Ignore any interest she would have earned on the $3000 she would have withdrawn from her savings account.

**ii** Calculate the interest savings over the 3-year period if Option 2 is adopted but now take into account the interest Kelly would have earned by leaving her $3000 in her savings account. Interest is paid at a rate of 3% per annum compounded monthly.

**6** **a** If Alex invests $1000 at 12.5% per annum simple interest, how long (in years) would it take before she has earned $1000 interest?

**b** If Alex invests $1000 at 12.5% per annum compound interest, compounded annually, how long would it take before she has earned $1000 interest? Give your answer to the nearest year.

**7** Janet wants to buy a CD player. The CD player she wants usually costs $1500, but is on sale for $1350.

**a** What percentage discount does this amount to?

**b** Janet considers entering into an agreement to buy the CD player where she pays no deposit and 12 monthly payments of $150.

**i** How much interest would Janet pay under this agreement on the purchase price of $1350?

**ii** What is the flat rate of interest per annum that this represents? Express your answer as a percentage correct to one decimal place.

**iii** What is the (approximate) effective interest rate that this represents? Express your answer as a percentage correct to one decimal place.

**8** Another shop offers the following plan on the CD player that Janet wants, but with a purchase price of $1500. Janet can have the CD player on no deposit and quarterly payments over 3 years. Interest will be charged at the rate of 8% per annum and compounded quarterly.

**a** Determine the quarterly repayment for this plan. Give your answer correct to the nearest cent.

**b** How much interest in total would be paid under this plan? Give your answer correct to the nearest cent.

**9** Geoff earns an annual salary of $44 200, which is paid fortnightly. He joined the superannuation fund on his 30th birthday and he pays 5% of his gross salary to the superannuation fund. His company contributes a further 10%.

**a** What amount of money is placed each fortnight into his superannuation fund?

**b** The superannuation fund pays 7% per annum compound interest, compounded fortnightly. Assuming that Geoff's annual salary remains constant, what is the amount of superannuation will Geoff have available at his 65th birthday?

c If there is an average of 4.5% inflation over the period of time that Geoff is working, what is the purchasing power of the amount of superannuation determined in part b?

d Suppose that when Geoff retires he places his superannuation in a perpetuity which will provide a monthly income without using any of the principal. If the perpetuity pays 6% per annum compounding monthly, what monthly payment will Geoff receive?

10 Helene has won $750 000 in a lottery. She decides to place the money in an investment account which pays 4.5% per annum interest, compounding monthly and to keep working and add another $1000 per month to this account.

a If she adheres to her plan for 10 years, how much will Helene have in the investment account?

b If there has been an average inflation rate of 2.2% over the ten-year period of her investment, what is the purchasing power of the amount of money Helene has in her account?

c After the ten years is up Helene decides to use her money to buy an annuity, which pays 3.5% per annum compounding monthly. If Helen requires $6000 per month for her living expenses, how long will the annuity last?

d Helene's accountant suggests that rather than purchase an annuity she places the money in a perpetuity so that she will be able to leave some money to her grandchildren. If the perpetuity pays 3.5% per annum compounding monthly, how much is the monthly payment that Helene will receive?