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CHAPTER

26

MODULE 6

Matrices and applications I

- What is a matrix?
- How do we add, subtract and multiply matrices?
- How do we use matrices to model real world situations?

26.1 What is a matrix?

The following table of data displays the heights, weights, ages and pulse rates of eight students.

Name	*Height*	*Weight*	*Age*	*Pulse rate*
Mahdi	173	57	18	86
Dave	179	58	19	82
Jodie	167	62	18	96
Simon	195	84	18	71
Kate	173	64	18	90
Pete	184	74	22	78
Mai	175	60	19	88
Tran	140	50	34	70

$$D = \begin{bmatrix} 173 & 57 & 18 & 86 \\ 179 & 58 & 19 & 82 \\ 167 & 62 & 18 & 96 \\ 195 & 84 & 18 & 71 \\ 173 & 64 & 18 & 90 \\ 184 & 74 & 22 & 78 \\ 175 & 60 & 19 & 88 \\ 140 & 50 & 34 & 70 \end{bmatrix}$$

If we extract the numbers from the table and enclose them in square brackets, we have formed a **matrix** (plural, matrices). We might call this matrix D (for data matrix). We use capital letters A, B, C, etc. to name matrices.

Rows and columns

Rows and columns are the building blocks of matrices. We number rows from the top down: Row 1, Row 2, etc. Columns are numbered from the left across: Column 1, Column 2, etc.

$$\begin{array}{rc}
 & \begin{array}{cccc} & & \text{Col. 3} & \end{array} \\
\begin{array}{r} \\ \text{Row 2} \\ \\ \\ D = \\ \\ \\ \\ \end{array} & \left[\begin{array}{cccc} 173 & 57 & 18 & 86 \\ 179 & 58 & 19 & 82 \\ 167 & 62 & 18 & 96 \\ 195 & 84 & 18 & 71 \\ 173 & 64 & 18 & 90 \\ 184 & 74 & 22 & 78 \\ 175 & 60 & 19 & 88 \\ 140 & 50 & 34 & 70 \end{array}\right]
\end{array}$$

Order of a matrix

In its simplest form, a matrix is just a rectangular array (rows and columns) of numbers. The **order** (or shape) of matrix D is said to be an (8×4) matrix, read '8 by 4'. This is because it has **eight rows** and **four columns**.

Order of a matrix

$$\text{order of a matrix} = \text{number of rows} \times \text{number of columns}$$

The numbers in the matrix are called **elements**. The number of elements in a matrix is determined by its order. For example, the number of elements in an (8×4) matrix is $8 \times 4 = 32$.

Row matrices

Matrices come in many shapes and sizes. For example, from this same set of data, we could have formed the matrix we might call K (for Kate's matrix):

$$K = \begin{bmatrix} 173 & 64 & 18 & 90 \end{bmatrix}$$

This matrix has been formed from just one row of the data, the data values for Kate. Because it contains just **one row** of numbers, it is called a **row matrix** (or **row vector**). It is a (1×4) matrix: one row by four columns. It contains $1 \times 4 = 4$ elements.

Column matrices

Equally, we could form a matrix we might call H (for height matrix). This matrix is formed from just one column of the data, the heights of the students.

Because it contains just **one column** of numbers, it is called a **column matrix** (or **column vector**). This is an (8×1) matrix: eight rows by one column. It contains $8 \times 1 = 8$ elements.

$$H = \begin{bmatrix} 173 \\ 179 \\ 167 \\ 195 \\ 173 \\ 184 \\ 175 \\ 140 \end{bmatrix}$$

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Square matrices

As a final example, we could form a matrix we might call M (for males). This matrix contains only the data for the males. As this matrix has four rows and four columns, it is a (4×4) matrix; four rows by four columns. It contains $4 \times 4 = 16$ elements.

$$M = \begin{bmatrix} 173 & 57 & 18 & 86 \\ 179 & 58 & 19 & 82 \\ 195 & 84 & 18 & 71 \\ 184 & 74 & 22 & 78 \end{bmatrix}$$

A matrix like M, with an **equal** number of **rows** and **columns** is called a **square matrix**.

Example 1 **Matrix facts**

For each of the matrices below, write down its type, order and the number of elements.

Solution

Matrix	Type	Order	No. of elements
$A = \begin{bmatrix} 1 & 5 & 1 \\ 2 & 0 & 4 \\ 2 & -1 & 6 \end{bmatrix}$	Square matrix no. of rows = no. of columns	(3×3) 3 rows, 3 cols.	9 $3 \times 3 = 9$
$B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$	Column matrix single column	(3×1) 3 rows, 1 col.	3 $3 \times 1 = 3$
$C = [3 \quad 1 \quad 0 \quad 5 \quad -3 \quad 1]$	Row matrix single row	(1×6) 1 row, 6 cols.	6 $1 \times 6 = 6$

Some notation

In some situations, we would like to talk about a matrix and its elements without having specific numbers in mind. We do this as follows.

For the matrix A, which has n rows and m columns, we write:

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \ldots & a_{1,m} \\ a_{2,1} & a_{2,2} & a_{2,3} & \ldots & a_{2,m} \\ a_{3,1} & a_{3,2} & a_{3,3} & \ldots & a_{3,m} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n,1} & a_{n,2} & a_{n,3} & \ldots & a_{n,m} \end{bmatrix}$$

$a_{2,3}$

row number column number

Thus:

- $a_{1,1}$ represents the element in the 1st row and the 1st column
- $a_{2,1}$ represents the element in the 2nd row and the 1st column
- $a_{1,2}$ represents the element in the 1st row and the 2nd column
- $a_{2,2}$ represents the element in the 2nd row and the 2nd column
- $a_{m,n}$ represents the element in the mth row and the nth column

Example 2 **Identifying the elements in a matrix**

For the matrices A and B, opposite, write down the values of:

a $a_{1,2}$ **b** $a_{2,1}$ **c** $a_{3,3}$ **d** $b_{3,1}$

$$A = \begin{bmatrix} 1 & 5 & 3 \\ -1 & 0 & 4 \\ 2 & -2 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

Solution

a $a_{1,2}$ is the element in the 1st row and the 2nd column of A $\qquad a_{1,2} = 5$

b $a_{2,1}$ is the element in the 2nd row and the 1st column of A $\qquad a_{2,1} = -1$

c $a_{3,3}$ is the element in the 3rd row and the 3rd column of A $\qquad a_{3,3} = 6$

d $b_{3,1}$ is the element in the 3rd row and the 1st column of B $\qquad b_{3,1} = 1$

Entering a matrix into a graphics calculator

Later in this chapter, you will learn about matrix arithmetic: how to add, subtract and multiply matrices. While it is possible to carry out these tasks by hand, for all but the smallest matrices this is extremely tedious. Most matrix arithmetic is better done with the help of a graphics calculator. However, before you can perform matrix arithmetic, you will need to know how to enter a matrix into your calculator. This will also give you practice with the subscript notation used by the graphics calculator.

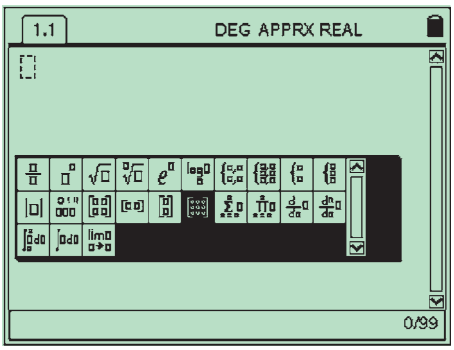
How to enter a matrix on the TI-Nspire CAS

Enter the matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$.

Steps

1 Start a new document and select **1:Calculator**.

2 Press ctrl + (templates key) to select the **Math Templates**. Use the cursor ▼ ▶ arrows to highlight the matrix template shown. Press enter.



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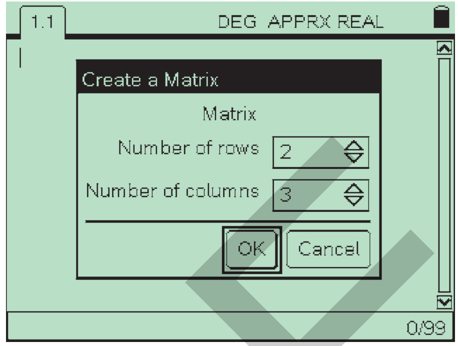
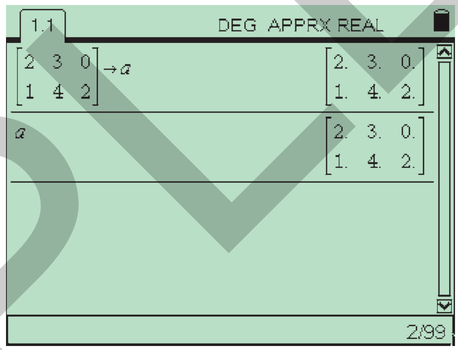
3 Use the ▼ arrow to select the **Number of rows** required. (In this case, the number of rows is 2.) Press (tab) to move to the next entry and repeat for the **Number of columns**. (In this case, the number of columns is 3.) Use (tab) to highlight **OK** and press (enter).



4 Type the values into the matrix template. Use (tab) to move to the required position in the matrix to enter each value.

When the matrix has been completed press (tab) to move outside the matrix and press (ctrl) + (sto▸ var), followed by (A). This will store the matrix as the variable a. Press (enter) .



5 When you type A (or a) in the graphics calculator it will paste in the matrix $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$. Press (enter) to display.

How to enter a matrix using the ClassPad

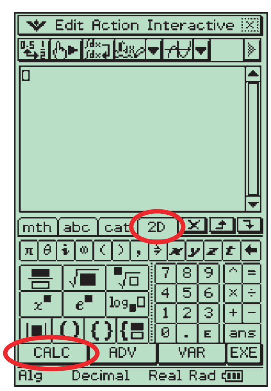
Enter the matrix $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$.

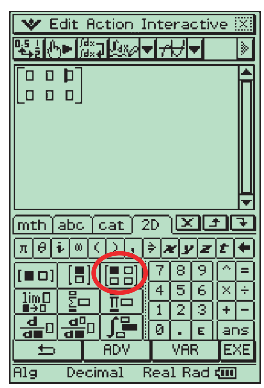
Steps

1 a From the application menu screen, locate and open the **Main** (Main) application. Press (Keyboard) to display the hidden keyboard.

b On the keyboard, tap the (2D) tab, followed by the (CALC) menu item at the bottom of the keyboard.

2 Tap the 2 ×2 matrix icon, followed by the 1 × 2 matrix icon. This will add a third column and create a 2 ×3 matrix.



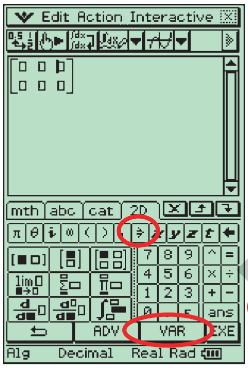


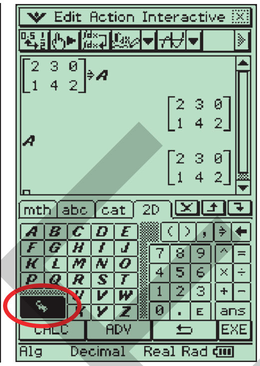
3 Enter the values of $\begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$.

Note: Tap at each new position to enter the new value.

Move the cursor to the very right-hand side of the matrix.

4 Assign the matrix the variable name $\boldsymbol{A}$. From the keyboard, tap the variable assignment key ⇒ (which can be found in the top row under the **2D** tab), followed by the VAR menu item (bottom of the keyboard), then CAP (for uppercase letters) and $\boldsymbol{A}$. Press EXE to confirm your choice. Until it is reassigned, $\boldsymbol{A}$ will represent the matrix as defined above.





Exercise 26A

1 Complete the sentences below that relate to the following matrices:

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 6 \\ -1 & 0 \\ 1 & 3 \\ 4 & -4 \end{bmatrix} \quad E = \begin{bmatrix} 4 & 3 & 1 \\ 0 & -1 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

a The square matrices are $\square$ and $\square$.

b Matrix B has $\square$ rows.

c The row matrix is $\square$.

d The column matrix is $\square$.

e Matrix D has $\square$ rows and $\square$ columns.

f The order of matrix E is $\square \times \square$.

g The order of matrix A is $\square \times \square$.

h The order of matrix B is $\square \times \square$.

i The order of matrix D is $\square \times \square$.

j There are $\square$ elements in matrix E.

k There are $\square$ elements in matrix A.

l $a_{1,4} = \square$ **m** $b_{3,1} = \square$ **n** $c_{1,1} = \square$ **o** $d_{4,1} = \square$ **p** $e_{2,2} = \square$

q $d_{3,2} = \square$ **r** $b_{1,1} = \square$ **s** $c_{1,2} = \square$ **t** $a_{1,2} = \square$ **u** $e_{1,3} = \square$

2 Enter the following matrices into a graphics calculator and display on the Home screen.

a $B = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{bmatrix}$ **b** $C = \begin{bmatrix} 4 & -4 \\ -2 & 6 \end{bmatrix}$ **c** $E = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$ **d** $F = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

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26.2 Using matrices to represent information

At the start of this chapter we used a matrix to store the numerical information in a data table. Matrices can also be used to carry codes that encrypt credit-card numbers for internet transmission or to carry all the information needed to solve sets of simultaneous equations. A less obvious application is to use matrices to represent network diagrams.

Example 3 Representing information in a table by a matrix

The table opposite shows the three types of membership of a local gym and the number of males and females enrolled in each. Construct a matrix to display the numerical information in the table.

	Gym membership		
Gender	*Weights*	*Aerobics*	*Fitness*
Males	16	104	86
Females	75	34	94

Solution

1 Draw in a blank (2 × 3) matrix. Label the rows M for male and F for female. Label the columns W for weights, A for aerobics and F for fitness.

$$\begin{array}{c c} & \begin{array}{ccc} W & A & F \end{array} \\ \begin{array}{c} M \\ F \end{array} & \begin{bmatrix} & & \\ & & \end{bmatrix} \end{array}$$

2 Fill in the elements of the matrix row by row, starting at the top left-hand corner of the table.

$$\begin{array}{c c} & \begin{array}{ccc} W & A & F \end{array} \\ \begin{array}{c} M \\ F \end{array} & \begin{bmatrix} 16 & 104 & 86 \\ 75 & 34 & 94 \end{bmatrix} \end{array}$$

Example 4 Entering a credit-card number into a matrix

Convert the 16-digit credit-card number: 4454 8178 1029 3161 into a 2 × 8 matrix.

Solution

1 Write out the sequence of numbers.

Note: Writing the number down in groups of four (as on the credit card) helps you keep track of the figures.

4454 8178 1029 3161

2 Form a 2 × 8 matrix, listing the digits in pairs, one under the other.

$$\begin{bmatrix} 4 & 5 & 8 & 7 & 1 & 2 & 3 & 6 \\ 4 & 4 & 1 & 8 & 0 & 9 & 1 & 1 \end{bmatrix}$$

A less obvious application is the use of matrices to represent information contained in network diagrams. Network diagrams are a series of numbered or labelled points joined in various ways. They provide a simplified way of representing and studying things as different as friendship networks, airline routes, electrical circuits and road links between towns. They are studied in detail in Module 5.

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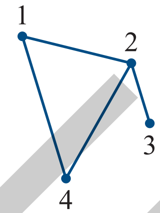
Example 5 Representing a network diagram by a matrix

Represent the network diagram shown opposite by a 4×4 matrix A, where the:

- matrix element $= 1$ if the two points are joined by a line.
- matrix element $= 0$ if the two points are not connected.

Note: Elements are the numbers in the matrix.



Solution

1 Draw in a blank 4×4 matrix, labelling the rows and columns 1, 2, 3, 4 to indicate the points.

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \end{array}\right] \end{array}$$

2 Fill in the elements of the matrix row by row, starting at the left-hand top corner:

- $a_{1,1} = 0$ (there is no line joining point 1 to itself)
- $a_{1,2} = 1$ (there is a line joining points 1 and 2)
- $a_{1,3} = 0$ (there is no line joining points 1 and 3)
- $a_{1,4} = 1$ (there is a line joining points 1 and 4)

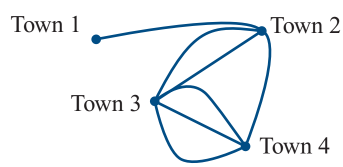
and so on until the matrix is complete.

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \end{array}$$

Note: If a network contains no 'loops' (lines joining points to themselves) the elements in the leading diagonal will always be zero. Knowing this can save a lot of work.

Example 6 Interpreting a matrix representing a network diagram

The diagram opposite shows the roads interconnecting four towns Town 1, Town 2, Town 3, and Town 4. This diagram has been represented by a 4×4 matrix, A. The elements show the number of roads between each pair of towns.



$$A = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 3 \\ 0 & 1 & 3 & 0 \end{array}\right] \end{array}$$

a In the matrix A, $a_{2,4} = 1$. What does this tell us?

b In the matrix A, $a_{3,4} = 3$. What does this tell us?

c In the matrix A, $a_{4,1} = 0$. What does this tell us?

d What is the sum of the elements in Row 3 of the matrix and what does this tell us?

e What is the sum of all the elements of the matrix and what does this tell us?

Solution

a There is one road between Town 2 and Town 4.

b There are three roads between Town 4 and Town 3.

c There is no road between Town 4 and Town 1.

d 5: The total number of roads between Town 3 and the other towns in the network.

e 14: The total number of different ways you can travel between towns.

Note: For each road, there are two ways you can travel, for example, from Town 1 to Town 2 ($a_{1,2} = 1$) and from Town 2 to Town 1 ($a_{2,1} = 1$).

Exercise 26B

1 The table of data opposite gives the number of residents, TVs and computers in three households. Use the table to:

	Residents	*TVs*	*Computers*
Household A	4	2	1
Household B	6	2	3
Household C	2	1	0

a construct a matrix to display the numerical information in the table. What is its order?

b construct a row matrix to display the numerical information in the table relating to Household *B*. What is its order?

c construct a column matrix to display the numerical information in the table relating to computers. What is its order? What does the sum of its elements tell you?

2 The table of data opposite gives the yearly car sales for two car dealers. Use the table to:

Car sales	*Small*	*Medium*	*Large*
Honest Joe's	24	32	11
Super Deals	32	34	9

a construct a matrix to display the numerical information in the table. What is its order?

b construct a row matrix to display the numerical information in the table relating to Honest Joe's. What is its order?

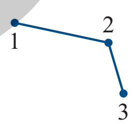
c construct a column matrix to display the numerical information in the table relating to small cars. What is its order? What does the sum of its elements tell you?

3 Convert the 16-digit credit-card number: 3452 8279 0020 3069 into a 2×8 matrix. List the digits in pairs, one under the other. Ignore spaces.

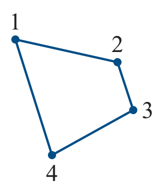
4 Represent each of the following network diagrams by a matrix A using the rules:

- matrix element = 1 if points are joined by a line
- matrix element = 0 if points are not joined by a line

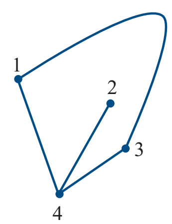
a



b



c



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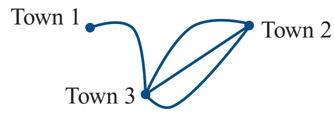
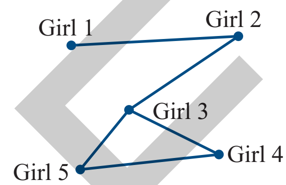
5 The diagram opposite shows the roads interconnecting three towns Town 1, Town 2 and Town 3.
Represent this diagram by a 3×3 matrix where the elements represent the number of roads between each pair of towns.



6 The network diagram opposite shows a friendship network between five girls, Girls 1 to 5.
This network has been represented by a 5×5 matrix F, using the rule:

- element = 1 if the pair of girls are friends
- element = 0 if the pair of girls are not friends



$$F = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a In the matrix F, $f_{3,4} = 1$. What does this tell us?
b In the matrix F, $f_{2,5} = 0$. What does this tell us?
c What is the sum of the elements in Row 3 of the matrix and what does this tell us?
d Which girl has the least friends? The most friends?

26.3 Matrix arithmetic: addition, subtraction and scalar multiplication

Equality of two matrices

Two matrices are equal if they are of the same order and each corresponding element is identical in value. It is not sufficient for the two matrices to contain an identical set of numbers; they must also be in the same positions.

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is equal to $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ because corresponding elements are equal

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is **not** equal to $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ same numbers but different positions

Matrix addition and subtraction

Adding and subtracting matrices
If two matrices are of the same order (same number of rows and columns), they can be added (subtracted) by adding (subtracting) their corresponding elements.

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Example 7 **Adding two matrices**

$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}$ Find $A + B$.

Solution

1 As the two matrices have the same order (2×3), they can be added.
2 Add corresponding elements.

$$A + B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2+0 & 3+2 & 0+3 \\ 1+2 & 4+(-2) & 2+1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 3 \\ 3 & 2 & 3 \end{bmatrix}$$

Likewise, if we have two matrices of the same order (same number of rows and columns), we can subtract the two matrices by subtracting their corresponding elements.

Example 8 **Subtracting two matrices**

$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}$ Find $A - B$

Solution

1 As the two matrices have the same order (2×3), they can be subtracted.
2 Subtract corresponding elements.

$$A - B = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 2-1 & 3-2 & 0-3 \\ 1-2 & 4-(-2) & 2-1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -3 \\ -1 & 6 & 1 \end{bmatrix}$$

Multiplying matrices by a number (scalar multiplication)

Mutiplying a matrix by a number has the effect of multiplying each element in the matrix by that number. Multiplying a matrix by a number is called **scalar multiplication**, because it has the effect of scaling the matrix by that number. For example, multiplying a matrix by 2 doubles each element in the matrix.

Example 9 **Scalar multiplication**

$A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 4 & -4 \\ -2 & 6 \end{bmatrix}$ Find $3A$ and $0.5C$.

Solution

Multiplying a matrix by a number has the effect of multiplying each element by that number.

$$3A = 3 \times \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 3\times 2 & 3\times 3 & 3\times 0 \\ 3\times 1 & 3\times 4 & 3\times 2 \end{bmatrix} = \begin{bmatrix} 6 & 9 & 0 \\ 3 & 12 & 6 \end{bmatrix}$$

$$0.5C = 0.5 \times \begin{bmatrix} 4 & -4 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 0.5\times 4 & 0.5\times(-4) \\ 0.5\times(-2) & 0.5\times 6 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$$

The zero matrix

If $X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $X - Y = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$.

A matrix of any order with **all zeros**, is known as a **zero matrix**. The symbol O is used to represent a zero matrix. The matrices below are all examples of zero matrices.

$$O = [0], \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad O = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Example 10 **The zero matrix**

$G = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix}$ $H = \begin{bmatrix} 9 & 0 \\ -6 & 3 \end{bmatrix}$ Show that $3G - 2H = O$.

Solution

$$3G - 2H = 3 \times \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix} - 2 \times \begin{bmatrix} 9 & 0 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 18 & 0 \\ -12 & 6 \end{bmatrix} - \begin{bmatrix} 18 & 0 \\ -12 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 18-18 & 0-0 \\ -12-(-12) & 6-6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore 3G - 2H = O$

Using a calculator to perform matrix addition, subtraction and scalar multiplication

For small matrices, it is usually quicker to add, subtract or multiply a matrix by a number (scalar multiplication) by hand. However, if dealing with larger matrices, it is best to use a graphics calculator.

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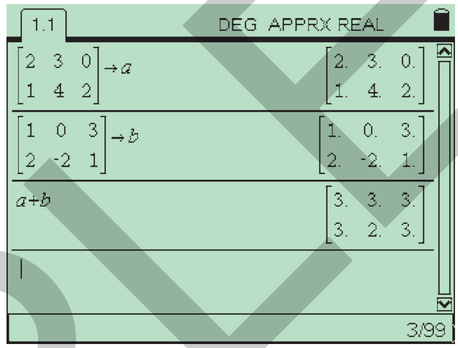
How to add, subtract and scalar multiply matrices using the TI-Nspire CAS

If $A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -2 & 1 \end{bmatrix}$, find:

a $A + B$ **b** $A - B$ **c** $3A - 2B$

Steps

1 Start a new document and select **1:Add Calculator**.

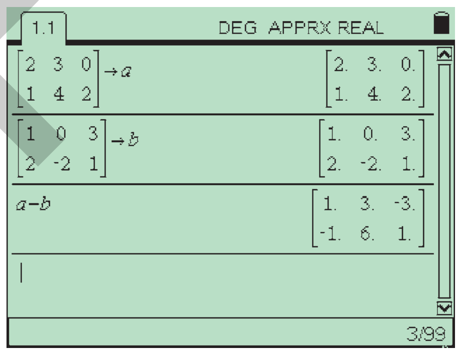


2 Enter the matrices A and B into your calculator.

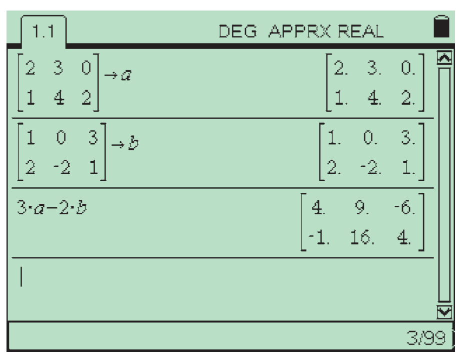
Note: Refer to page 639 if you are unsure of how to enter a matrix into your calculator.

a To determine $A + B$, type ***a* + *b***. Press enter to evaluate.

b To determine. $A - B$, type ***a* − *b***. Press enter to evaluate.



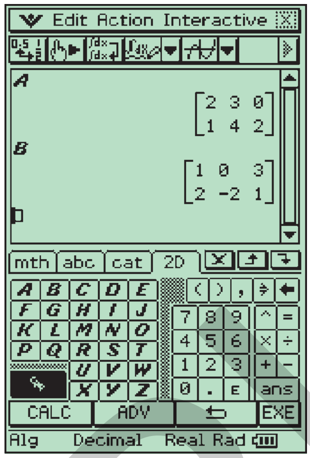
c To determine $3A - 2B$, type **3*a* + 2*b***. Press enter to evaluate.

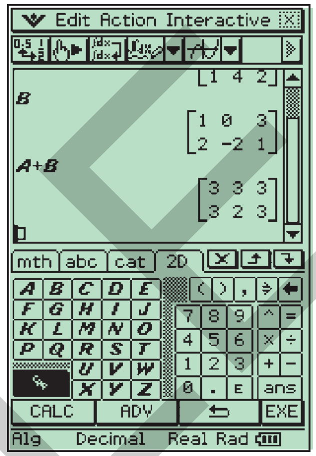


How to add, subtract and scalar multiply matrices with the ClassPad

1 Enter the matrices A and B into your calculator.
Note: Refer to page 639 if you are unsure of how to enter a matrix into your calculator.

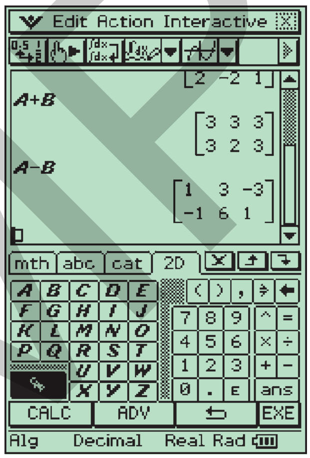
a To calculate $A + B$, type $\boldsymbol{A + B}$ and then press (EXE) to evaluate.

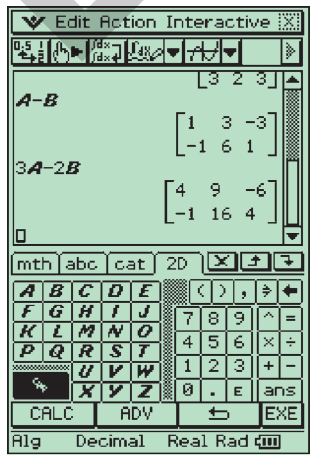




b To calculate $A + B$, type $\boldsymbol{A - B}$ and then press (EXE) to evaluate.

c To calculate $3A - 2B$, type $\boldsymbol{3A - 2B}$ and then press (EXE) to evaluate.





Example 11 Processing data using matrix addition, subtraction and scalar multiplication

The sales data for two small used car dealers, Honest Joe's and Super Deals, is displayed in the table below.

	2005			*2006*		
Car sales	*Small*	*Medium*	*Large*	*Small*	*Medium*	*Large*
Honest Joe's	24	32	11	26	38	16
Super Deals	32	34	9	35	41	12

Solution

a Construct two matrices A and B which represent the sales data for 2005 and 2006 separately

$$A = \begin{bmatrix} 24 & 32 & 11 \\ 32 & 34 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 26 & 38 & 16 \\ 35 & 41 & 12 \end{bmatrix}$$

b Construct a new matrix $C = A + B$. What does this matrix represent?

$$C = A + B = \begin{bmatrix} 24 & 32 & 11 \\ 32 & 34 & 9 \end{bmatrix} + \begin{bmatrix} 26 & 38 & 16 \\ 35 & 41 & 12 \end{bmatrix} = \begin{bmatrix} 50 & 70 & 27 \\ 67 & 75 & 21 \end{bmatrix}$$

Matrix C represents the total sales for 2005 and 2006 for the two dealers.

c Construct a new matrix $D = B - A$. What does this matrix represent?

$$D = B - A = \begin{bmatrix} 26 & 38 & 16 \\ 35 & 41 & 12 \end{bmatrix} - \begin{bmatrix} 24 & 32 & 11 \\ 32 & 34 & 9 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 5 \\ 3 & 7 & 3 \end{bmatrix}$$

Matrix D represents the increase in sales from 2005 and 2006 for the two dealers.

d Both dealers want to increase their 2006 sales, by 50% by 2009. Construct a new matrix $E = 1.5B$. Explain why this matrix represents the planned sales figures for 2009.

$$E = 1.5B = 1.5 \times \begin{bmatrix} 26 & 38 & 16 \\ 35 & 41 & 12 \end{bmatrix} = \begin{bmatrix} 1.5 \times 26 & 1.5 \times 38 & 1.5 \times 16 \\ 1.5 \times 35 & 1.5 \times 41 & 1.5 \times 12 \end{bmatrix} = \begin{bmatrix} 39 & 57 & 24 \\ 52.5 & 61.5 & 18 \end{bmatrix}$$

Forming the scalar product 1.5B multiplies each element by 1.5 which has the effect of increasing each value by 50%.

Skillsheet TEACHER

Exercise 26C

1 The questions below relate to the following six matrices. Computations will be quicker if done by hand.

$$A = \begin{bmatrix} 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \quad F = \begin{bmatrix} 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

a Which matrices are equal?

b Which matrices have the same order?

c Which matrices can be added or subtracted?

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d Compute each of the following, where possible:

i $A + B$ **ii** $D + E$ **iii** $C - F$ **iv** $A - B$ **v** $E - D$

vi $3B$ **vii** $4F$ **viii** $3C + F$ **ix** $4A - 2B$ **x** $E + F$

2 a $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} =$ **b** $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} =$ **c** $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} + 2 \times \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} =$

d $\begin{bmatrix} 1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \end{bmatrix} =$ **e** $\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} =$ **f** $3\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} =$

g $\begin{bmatrix} 2 \\ 0 \end{bmatrix} + 2[0 \quad 2] =$ **h** $3\begin{bmatrix} 0 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} =$ **i** $\begin{bmatrix} 2 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & -2 \end{bmatrix} =$

j $\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} =$

3 Use a calculator to evaluate the following:

a $2.2 \times \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} - 1.1 \times \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} =$ **b** $\begin{bmatrix} 1.2 & 0.2 \\ 4.5 & 3.3 \end{bmatrix} - 3.5 \times \begin{bmatrix} 0.4 & 4 \\ 1 & 2 \end{bmatrix} =$

c $5 \times \begin{bmatrix} 1 & 2 & 1 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} - 2 \times \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 1 \\ 0.5 & 0 & -2 \end{bmatrix} =$

d $0.8 \times \begin{bmatrix} 1 & 2 & 1 & 4 \\ 1 & 0 & -1 & 2 \end{bmatrix} + 0.2 \times \begin{bmatrix} -1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 2 \end{bmatrix} =$

4 The number of CDs sold in a company's city, suburban and country stores for each 3-month period in a year are shown in the table.

CD sales (thousands)	*Time of year*			
	Jan–March	*April–June*	*July–Sept*	*Oct–Dec*
City store	2.4	2.8	2.5	3.4
Suburban store	3.5	3.4	2.6	4.1
Country store	1.6	1.8	1.7	2.1

a Construct four (3×1) matrices A, B, C, and D that show the sales in each of the three-month periods during the year.

b Evaluate $A + B + C + D$. What does the sum $A + B + C + D$ represent?

5 The numbers of females and males enrolled in three different gym programs, *Weights*, *Aerobics* and *Fitness*, for 2005 and 2006 are shown in the table.

Gym membership	*2005*			*2006*		
	Weights	*Aerobics*	*Fitness*	*Weights*	*Aerobics*	*Fitness*
Females	16	104	86	24	124	100
Males	75	34	94	70	41	96

a Construct two matrices A and B which represent the gym membership for 2005 and 2006 separately.

b Construct a new matrix $C = A + B$. What does this matrix represent?

c Construct a new matrix $D = B - A$. What does this matrix represent? What do the negative elements in this matrix represent?

d The manager of the gymnasium wants to double her 2006 membership by 2009. Construct a new matrix E that would show the membership in 2009 if she succeeds with her plan. Evaluate.

26.4 Matrix arithmetic: the product of two matrices

The process of multiplying two matrices involves both multiplication and addition. It can be illustrated using Australian rules football scores.

An illustration of matrix multiplication

Two teams, the Ants and the Bulls play each other. At the end of the game:

- the Ants had scored 11 goals 5 behinds
- the Bulls had scored 10 goals 9 behinds

Now calculate each team's score in points:

- one goal = 6 points
- one behind = 1 point.

Thus we can write:

$$11 \times 6 + 5 \times 1 = 71 \text{ points}$$

$$10 \times 6 + 9 \times 1 = 69 \text{ points}$$

Matrix multiplication follows the same pattern.

$$\begin{array}{r c c c c c} & \begin{array}{cc}\text{Goals} & \text{Behinds}\end{array} & & \text{Point values} & & \text{Final points} \\ \begin{array}{r}\text{Ants score:}\\ \text{Bulls score:}\end{array} & \begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix} & \times & \begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \times 6 + 5 \times 1 \\ 10 \times 6 + 9 \times 1 \end{bmatrix} & = & \begin{bmatrix} 71 \\ 69 \end{bmatrix} \end{array}$$

The order of matrices and matrix multiplication

Look at the **order** of each of the matrices involved in the matrix multiplication below.

$$\begin{array}{r c c c c c} & \begin{array}{cc}\text{Goals} & \text{Behinds}\end{array} & & \text{Point values} & & \text{Final points} \\ \begin{array}{r}\text{Ants score:}\\ \text{Bulls score:}\end{array} & \begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix} & \times & \begin{bmatrix} 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \times 6 + 5 \times 1 \\ 10 \times 6 + 9 \times 1 \end{bmatrix} & = & \begin{bmatrix} 71 \\ 69 \end{bmatrix} \\ \text{Order of matrices:} & (2 \times 2) & & (2 \times 1) & & (2 \times 1) \end{array}$$

Thus, multiplying a (2×2) matrix by a (2×1) matrix gives a (2×1) matrix.

Two observations can be made here:

- To perform matrix multiplication, the **number of columns in the first matrix** (2) need to be the same as the **number of rows in the second matrix** (2). For example, if there were three columns in the first matrix, there would not be enough elements in the second matrix

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to complete the multiplication. When this happens, we say that matrix multiplication is not defined.

- The final result of multiplying the two matrices is a (2×1) matrix. For each row in the first matrix, there will be a row in the product matrix (there are 2 rows). For each column in the second matrix, there will be a column in the product matrix (there is 1 column).

These observations can be generalised to give two important rules for matrix multiplication.

Rule 1: Condition for matrix multiplication to be defined

Matrix multiplication of two matrices requires the **number of columns** in the **first matrix** to **equal** the **number of rows** in the **second matrix**.

That is, if A is of order $(m \times n)$ and B is of order $(r \times s)$, then the product AB is only defined if $n = r$.

For example, if $A = \begin{bmatrix} 2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$, then:

- $AB = \underset{(2 \times 3)}{\begin{bmatrix} 2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}} \times \underset{(3 \times 1)}{\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}}$ is **defined**: the number of columns in A (3) = the number of rows in B (3)
- $BC = \underset{(3 \times 1)}{\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}} \times \underset{(2 \times 2)}{\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}}$ is **not defined**: the number of columns in B (1) $\neq$ the number of rows in C (2)

Example 12 **Is a matrix product defined?**

$A = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Which of the following matrix products are defined?

a AB **b** BC **c** AC

Solution

1 Write down the matrix product. Under each matrix, write down its order (no. of rows × no. of columns).

2 The matrix product is defined if: no. of columns in Matrix 1 = no. of rows in Matrix 2.

3 Write down your conclusion.

a $AB = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \end{bmatrix}$; not defined

order: (2×2) (1×2)

b $BC = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; defined

order: (1×2) (2×1)

c $AC = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; defined

order: (2×2) (2×1)

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Once we know that two matrices can be multiplied, we can use the order of the two matrices to determine the order of the resulting matrix.

Rule 2: Determining the order of the product matrix

If two matrices can be multiplied, then the **product matrix** will have the same **number of rows** as the **first matrix** and the same **number of columns** as the **second matrix**.

That is, if A is of order $(m \times n)$ and B is of order $(n \times s)$, then AB will be a matrix of order $(m \times s)$.

For example, if $A = \begin{bmatrix} 2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$, then:

- $AB = \begin{bmatrix} 2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ is **defined** and will be of order (2×1)

  $(2 \times \underbrace{3) \quad (3}_{\text{equal}} \times 1) \qquad (2 \times 1)$

- $CA = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$ is **defined** and will be of order (2×3)

  $(2 \times \underbrace{2) \quad (2}_{\text{equal}} \times 3) \qquad (2 \times 3)$

Example 13 Determining the order of a matrix product

$$A = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

The following matrix products are defined. What is their order?

a BA **b** BC **c** AC

Solution

1 Write down the matrix product. Under each matrix, write down its order (no. of rows × no. of columns).

2 The order of the product matrix is given by (no. of rows in Matrix 1× no. of columns in Matrix 2).

3 Write down the order.

a $BA = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix}$; order of BA: (1×2)

order: $(1 \times 2)\ (2 \times 2)$

b $BC = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; order of BC: (1×1)

order: $(1 \times 2)\ (2 \times 1)$

c $AC = \begin{bmatrix} 6 & 0 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; order of AC: (2×1)

order: $(2 \times 2)\ (2 \times 1)$

Order of multiplication is important when multiplying matrices

You might have noticed in Example 13 that while the matrix product BA was defined, the matrix product AB in Example 12 was not defined. Order is important in matrix multiplication. For example, if we have two matrices, M and N, and form the products MN and NM, frequently the products will be different. We will return to this point when we learnt how to determine matrix products.

Determining matrix products

The process of matrix multiplication is a complex and extremely error prone and tedious process to do by hand. Fortunately, graphics calculators will do it for us, and that is perfectly acceptable. However, before we show you how to use a graphics calculator to multiply matrices, we will illustrate the process by multiplying a row matrix by a column matrix by hand.

Example 14 **Multiplying a row matrix by a column matrix**

Evaluate the matrix product AB, where $A = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$.

Solution

1 Write down the matrix product and, above each matrix, write down its order. Use this information to determine whether the matrix product is defined and its order.

$$AB = \overset{(1 \times 3)}{\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}} \overset{(3 \times 1)}{\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}}$$

AB is defined because the number of columns in A equals the number of rows in B.
The order of AB is (1×1).

2 To determine the matrix product:
  i multiply each element in the row matrix by the corresponding element in the column matrix
  ii add the results
  iii write down your answer

$$\begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = [1 \times 2 + 3 \times 4 + 2 \times 1] = [16]$$

$$\therefore AB = [16]$$

Example 15 **Multiplying a square matrix by a column matrix**

Evaluate the matrix product AB, where $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

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Solution

1 Write down the matrix product and, above each matrix, write down its order. Use this information to determine whether the matrix product is defined and its order.

$$\begin{array}{ccc} & (2 \times 2) & (2 \times 1) \\ AB = & \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \end{bmatrix} \end{array}$$

AB is defined because the number of columns in A equals the number of rows in B.
The order of AB is (2×1).

2 To determine the matrix product:

- **i** multiply each element in the row matrix by the corresponding element in the column matrix
- **ii** add the results
- **iii** write down your answer

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \times 2 + 0 \times 3 \\ 2 \times 2 + 3 \times 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 13 \end{bmatrix}$$

$$\therefore AB = \begin{bmatrix} 2 \\ 13 \end{bmatrix}$$

In principle, if you can multiply a row matrix by a column matrix, you can work out the product between any two matrices, provided it is defined. However, because you have to do it for every possible row/column combination it soon gets beyond the most patient and careful human being. For that reason, in practice we make use of technology to do it for us.

Using a calculator to multiply two matrices

We will illustrate how to use a calculator to multiply matrices by evaluating the matrix product in the football score example given earlier.

How to multiply two matrices using the TI-Nspire CAS

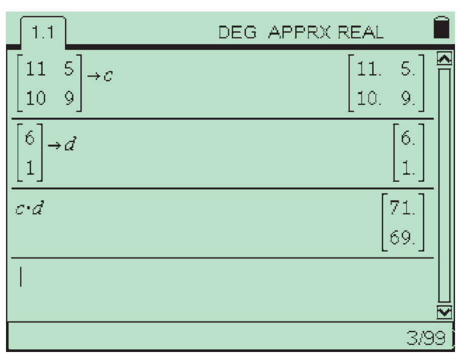
If $C = \begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix}$ and $D = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, find the matrix product CD.

Steps

1 Start a new document and select **1:Add Calculator**.

2 Enter the matrices C and D into your calculator.

3 To calculate matrix CD, type $\boldsymbol{c} \times \boldsymbol{d}$. Press (enter) to evaluate.
Note: You must put a multiplication sign between the c and d.

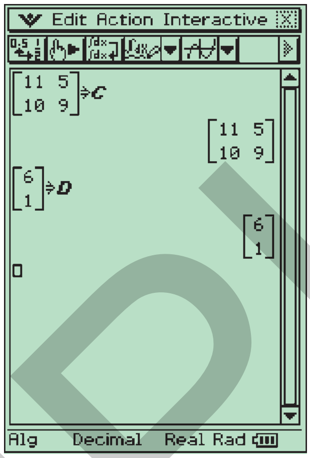


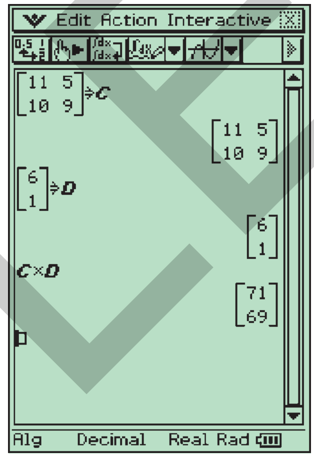
How to multiply two matrices with the ClassPad

If $C = \begin{bmatrix} 11 & 5 \\ 10 & 9 \end{bmatrix}$ and $D = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$, find the matrix product CD.

Steps

1. Enter the matrices C and D into your calculator.
2. To calculate CD, type **C** $\times$ **D** and then press (EXE) to evaluate.





Applications of the product of two matrices

Example 16 A practical application of matrix multiplication

$E = \begin{array}{c} \begin{bmatrix} 25 \\ 40 \end{bmatrix} \end{array} \begin{array}{l} \textit{Walk} \\ \textit{Run} \end{array}$ Matrix E gives the energy in kilojoules consumed per minute when walking and jogging

$T = \begin{array}{cc} \begin{bmatrix} 20 & 40 \end{bmatrix} \\ \textit{Walk} \quad \textit{Run} \end{array}$ Matrix T gives the times (in minutes) a person spent walking and running in a training session.

Compute the matrix product TE and show that it gives the total energy consumed during the training session.

Solution

$$TE = \begin{bmatrix} 20 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 40 \end{bmatrix} = [20 \times 25 + 40 \times 40] = [2100]$$

The total energy consumed is:

20 minutes $\times$ 25 kJ/minute + 40 minutes $\times$ 40 kJ/minute = 2100 kJ

This is the value given by the matrix product TE.

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You could, of course, work out the energy consumed on the training run for one person just as quickly without using matrices. However, the advantage of using a matrix formulation is that, with the aid of a calculator, you could have almost as quickly worked out the energy consumed by ten or more different runners, all with different times spent walking and running.

In the next chapter, you will learn to solve sets of simultaneous linear equations using matrices. The first step in this process is to write a set of simultaneous equations in matrix notation. This process involves the use of matrix multiplication and serves as our next example.

Example 17 **Generating sets of simultaneous equations**

Show that the matrix equation $\begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ generates the following pair of simultaneous linear equations: $\begin{matrix} 4x + 2y = 5 \\ 3x + 2y = 2 \end{matrix}$

Solution

1 Write out the matrix equation, then multiply out the left-hand side.

$$\begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 4 \times x + 2 \times y \\ 3 \times x + 2 \times y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\text{or } \begin{bmatrix} 4x + 2y \\ 3x + 2y \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

2 For the matrices to be equal:

$4x + 2y$ must equal 5

and

$3x + 2y$ must equal 2.

This gives the required pair of simultaneous linear equations.

$$\therefore \begin{matrix} 4x + 2y = 5 \\ 3x + 2y = 2 \end{matrix}$$

Exercise 26D

Skillsheet TEACHER

1 The questions below relate to the following six matrices.

$$A = \begin{bmatrix} 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \quad F = \begin{bmatrix} 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

a Which of the following matrix products are defined?

i AB **ii** BA **iii** AC **iv** CE **v** EC **vi** EF **vii** FE **viii** FA

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b Compute the following products by hand.

i AB **ii** CE **iii** DB **iv** FE

c Enter the six matrices into your calculator and compute the following matrix expressions.

i AB **ii** FE **iii** $AB - 3CE$ **iv** $2FE + 3B$

2 By hand, or using a calculator, evaluate each of the following matrix products.

a $\begin{bmatrix} 0 & 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ 0 \end{bmatrix} =$ **b** $\begin{bmatrix} 10 & -30 \end{bmatrix} \times \begin{bmatrix} 5 \\ 1 \end{bmatrix} =$ **c** $\begin{bmatrix} -2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} =$

d $\begin{bmatrix} 2 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 100 \\ 200 \\ 0 \end{bmatrix} =$ **e** $\begin{bmatrix} 0 & 2 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix} =$

f $\begin{bmatrix} 0 & 0.2 & 0.1 & 0.3 \end{bmatrix} \times \begin{bmatrix} 10 \\ 20 \\ 30 \\ 20 \end{bmatrix} =$ **g** $\begin{bmatrix} 0.5 \\ -1.5 \\ 2.5 \end{bmatrix} \times \begin{bmatrix} 2 & 4 & 6 \end{bmatrix} =$

h $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} =$ **i** $\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix} \times \begin{bmatrix} 0 & 3 \\ 1 & 2 \end{bmatrix} =$

j $\begin{bmatrix} 10 & -10 \\ 40 & 30 \end{bmatrix} \times \begin{bmatrix} 0 & 0.3 \\ 0.1 & 0.2 \end{bmatrix} =$ **k** $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} =$

l $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ -1 & 2 \end{bmatrix} =$ **m** $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 2 & 1 \\ -1 & 4 & 2 \\ -2 & 1 & 2 \end{bmatrix} =$

3 Six teams play an indoor soccer competition. If a team:

- wins, they score two points
- draws the game, they score one point
- loses, they score zero points

This is summarised in the points matrix opposite.

$$P = \begin{array}{c} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} Win \\ Draw \\ Lose \end{array}$$

The results of the competition are summarised in the results matrix.

Work out the final points score for each team, by forming the matrix product RP.

$$R = \begin{array}{c} \begin{array}{ccc} W & D & L \end{array} \\ \begin{bmatrix} 4 & 1 & 0 \\ 3 & 1 & 1 \\ 3 & 0 & 2 \\ 1 & 2 & 2 \\ 1 & 1 & 3 \\ 0 & 1 & 4 \end{bmatrix} \end{array} \begin{array}{l} \\ Team1 \\ Team2 \\ Team3 \\ Team4 \\ Team5 \\ Team6 \end{array}$$

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4 Four people complete a training session in which they walked, jogged and ran at various times.

The energy consumed in kJ/minute when walking, jogging or running is listed in the energy matrix opposite.

$$E = \begin{array}{cc} \begin{bmatrix} 25 \\ 40 \\ 65 \end{bmatrix} & \begin{array}{l} Walk \\ Jog \\ Run \end{array} \end{array}$$

The time spent in each activity (in minutes) by four people is summarised in the time matrix opposite. Work out the total energy consumed by each person, by forming the matrix product *TE*.

$$T = \begin{array}{cc} \begin{array}{ccc} W & J & R \end{array} & \\ \begin{bmatrix} 10 & 20 & 30 \\ 15 & 20 & 25 \\ 20 & 20 & 20 \\ 30 & 20 & 10 \end{bmatrix} & \begin{array}{l} Person\ 1 \\ Person\ 2 \\ Person\ 3 \\ Person\ 4 \end{array} \end{array}$$

5 Show, by multiplying out the matrices, that the matrix equation:

$$\begin{bmatrix} 1 & 3 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 5 \end{bmatrix}$$ generates the pair of equations: $$\begin{aligned} x + 3y &= 16 \\ 2x - 4y &= 5 \end{aligned}$$

6 Show, by multiplying out the matrices, that the matrix equation:

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$$ generates the set of equations: $$\begin{aligned} 2x + y + 3z &= 2 \\ 3x + 2y - z &= 4 \\ 2x \quad + 3z &= 3 \end{aligned}$$

MC TEST D&D TEST

Review

Key ideas and chapter summary

Matrix (plural: matrices)
A **matrix** is a rectangular array of numbers or symbols (elements) enclosed in brackets.

$[5]$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $[2 \quad 0 \quad 1]$ $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$

are all examples of matrices.

Row matrix (row vector)
A **row matrix** contains a **single row** of elements.
$[2 \quad 0 \quad 1]$ is an example of a row matrix.

Column matrix (column vector)
A **column matrix** contains a **single column** of elements.
$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is an example of a column matrix.

Square matrix
A **square** matrix has an **equal number of rows and columns**.
$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ is an example of a square matrix.

The null (zero) matrix
A **null** (**zero**) matrix contains only zeros.
$[0]$ $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $[0 \quad 0 \quad 0]$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are all examples of null matrices.

Order of a matrix
The **order** of a matrix is defined by the number of rows and columns.
A matrix with m rows and n columns is said to be of order $m \times n$ (read 'm by n').
For example:
$[2 \quad 0 \quad 1]$ is a (1×3) matrix: one row and three columns
$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ is a (2×2) matrix: two rows and two columns
The order of a matrix is important in determining whether it can be added to, subtracted from or multiplied by another matrix.

Locating an element in a matrix
The location of each element in the matrix is specified by its row and column number.

For example, in the matrix $A = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix}$ the element

- $a_{1,2} = -1$ is in the 1st row and 2nd column
- $a_{2,1} = 4$ is in the 2nd row and 1st column

For a $(m \times n)$ matrix, the **number of elements** $= \mathbf{m \times n}$.
For example:
$[2 \quad 0 \quad 1]$ is a (1×3) matrix and has $1 \times 3 = 3$ elements
$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$ is a (2×2) matrix and has $2 \times 2 = 4$ elements.

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Equality of matrices

Matrices are **equal** when they have the **same order** and **corresponding elements** are **equal** in value.

For example:

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \text{ but } \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \text{ even though the}$$

values of all the elements are equal.

Adding and subtracting matrices

Two matrices of the same order can be added or subtracted, by adding or subtracting, corresponding elements.

For example:

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2+(-1) & 0+1 \\ 1+2 & 1+3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2-(-1) & 0-1 \\ 1-2 & 1-3 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & -2 \end{bmatrix}$$

Scalar multiplication

Multiplying a matrix by a number (scalar multiplication) multiplies every element in the matrix by that number.

For example:

$$3\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3\times 2 & 3\times 0 \\ 3\times 1 & 3\times 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 3 & 3 \end{bmatrix}$$

Matrix multiplication

- If A is of order $(m \times n)$ and B is of order $(r \times s)$, then the product AB:
  - is only defined if $n = r$
  - will be a matrix of order $(m \times s)$

  For example:

  $$\underset{(2\times 2)}{\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}} \times \underset{(2\times 1)}{\begin{bmatrix} 2 \\ -1 \end{bmatrix}} \text{ is **defined** and will be of order } (2\times 1)$$

  $$\underset{(3\times 2)}{\begin{bmatrix} 3 & 0 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}} \times \underset{(2\times 3)}{\begin{bmatrix} 2 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}} \text{ is **defined** and will be of order } (3\times 3)$$

- The order in which two matrices are multiplied is important.

  For example:

  $$\underset{(2\times 2)}{\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}} \times \underset{(2\times 1)}{\begin{bmatrix} 2 \\ -1 \end{bmatrix}} \text{ is **defined** but } \underset{(2\times 1)}{\begin{bmatrix} 2 \\ -1 \end{bmatrix}} \times \underset{(2\times 2)}{\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}} \text{ is **not defined**.}$$

- Matrix multiplication is a process of multiplying rows by columns. To multiply a row matrix by a column matrix, each element in the row matrix is multiplied by each element in the column matrix and the results added.

  For example:

$$\begin{bmatrix}1 & 0 & 3\end{bmatrix}\begin{bmatrix}4\\2\\5\end{bmatrix} = [1 \times 4 + 0 \times 2 + 3 \times 5] = [19]$$

- When multiplying matrices with more that one row or column, the process is repeated, until all possible row and column combinations have been exhausted.

$$\begin{bmatrix}1 & 0\\2 & 3\end{bmatrix}\begin{bmatrix}4\\2\end{bmatrix} = \begin{bmatrix}1 \times 4 + 0 \times 2\\2 \times 4 + 3 \times 2\end{bmatrix} = \begin{bmatrix}4\\14\end{bmatrix}$$

Skills check

Having completed this chapter you should be able to:

- determine the order of a matrix
- recognise a row, column and square matrix
- recognise a (zero) matrix
- know when matrices can be added, subtracted and multiplied
- when appropriate, add or subtract two matrices
- multiply a matrix by a number (scalar multiplication)
- when appropriate, multiply two matrices
- solve application problems involving matrices

Multiple-choice questions

The following matrices are needed for Questions 1 to 7

$$U = \begin{bmatrix}2 & 0\\1 & 1\end{bmatrix} \quad V = \begin{bmatrix}-1\\1\end{bmatrix} \quad W = \begin{bmatrix}1 & -1\end{bmatrix} \quad X = \begin{bmatrix}3 & 1 & -1\\1 & 0 & 1\end{bmatrix}$$

$$Y = \begin{bmatrix}0 & 1\\-1 & 2\end{bmatrix} \quad Z = \begin{bmatrix}1\\0\\1\end{bmatrix}$$

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1 The row matrix is:

A U **B** V **C** W **D** X **E** Z

2 The square matrices are:

A U and V **B** X and Y **C** Y and W **D** U and Y **E** U, V and X

3 The order of matrix X is:

A (2×2) **B** (2×3) **C** (3×2) **D** (3×3) **E** (6)

4 The following matrices can be added:

A U and V **B** V and W **C** X and Y **D** U and Y **E** none of the above

5 The following matrix product is **not** defined:

A WV **B** XZ **C** YV **D** XY **E** UY

6 $-2Y =$

A $\begin{bmatrix} 0 & -2 \\ 2 & -4 \end{bmatrix}$ **B** $\begin{bmatrix} 0 & 2 \\ -2 & 4 \end{bmatrix}$ **C** $\begin{bmatrix} 0 & -2 \\ -2 & 4 \end{bmatrix}$ **D** $\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ **E** $\begin{bmatrix} -2 & -1 \\ -4 & 2 \end{bmatrix}$

7 The order of matrix product XZ is:

A (1×3) **B** (2×1) **C** (3×1) **D** (3×2) **E** (3×3)

8 In the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -1 & 3 \\ -5 & -4 & 7 \end{bmatrix}$ the element $a_{2,3} =$

A -4 **B** -1 **C** 0 **D** 3 **E** 4

9 $2\begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} =$

A $\begin{bmatrix} 5 & 0 \\ -4 & 0 \end{bmatrix}$ **B** $\begin{bmatrix} 5 & 0 \\ 4 & 0 \end{bmatrix}$ **C** $\begin{bmatrix} 3 & 0 \\ -2 & 0 \end{bmatrix}$ **D** $\begin{bmatrix} 6 & 0 \\ 1 & 2 \end{bmatrix}$ **E** $\begin{bmatrix} 5 & 0 \\ -3 & 1 \end{bmatrix}$

10 $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} =$

A $\begin{bmatrix} 10 \end{bmatrix}$ **B** $\begin{bmatrix} 12 \end{bmatrix}$ **C** $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ **D** $\begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \end{bmatrix}$ **E** not defined

11 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \end{bmatrix} =$

A $\begin{bmatrix} 18 \end{bmatrix}$ **B** $\begin{bmatrix} 14 \end{bmatrix}$ **C** $\begin{bmatrix} 3 \\ 7 \end{bmatrix}$ **D** $\begin{bmatrix} 4 \\ 10 \end{bmatrix}$ **E** $\begin{bmatrix} 2 & 2 \\ 6 & 4 \end{bmatrix}$

12 $\begin{bmatrix} 2 & 0 & 1 \\ 9 & -1 & 4 \\ 2 & 5 & 0 \end{bmatrix} \times \begin{bmatrix} -1 & 2 \\ 0 & 4 \end{bmatrix} =$

A $\begin{bmatrix} -2 & 2 \\ 6 & 0 \end{bmatrix}$ **B** $\begin{bmatrix} -2 & 2 \\ -6 & 0 \end{bmatrix}$ **C** $\begin{bmatrix} -2 & 0 & 6 \\ 3 & 0 & 7 \end{bmatrix}$

D $\begin{bmatrix} -2 & 0 & 6 \\ 2 & -1 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ **E** not defined

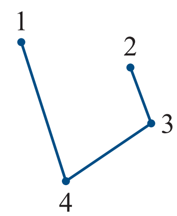
13 $\begin{bmatrix} 2 & 0 & 2 \\ 4 & -2 & 8 \\ 2 & 6 & 0 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0 & -1.5 \\ 0 & -1.5 & 0.5 \\ 0.5 & -1.5 & 0 \end{bmatrix} =$

A $\begin{bmatrix} -2 & 7 & -8 \end{bmatrix}$ **B** $\begin{bmatrix} -2 \\ 7 \\ -8 \end{bmatrix}$ **C** $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 3 & 4 \\ 1 & -9 & 0 \end{bmatrix}$

D $\begin{bmatrix} 2 & -3 & -3 \\ 6 & -9 & -7 \\ 1 & -9 & 0 \end{bmatrix}$ **E** not defined

14 The diagram opposite is to be represented by a matrix A, where:

- element $=$ 1 if the two points are joined by a line
- element $=$ 0 if the two points are not connected.



The matrix A is:

A $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$ **B** $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ **C** $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

D $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ **E** $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}$

15 The matrix equation $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ generates the following pair of simultaneous linear equations:

A $x + 4y = 2$
$3x - 2y = 1$

B $3x - 2y = 1$
$x + 4y = 2$

C $x + 2y = 1$
$4x + y = 2$

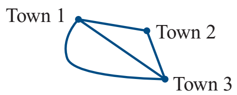
D $3x + 2y = 1$
$x - 4y = 2$

E $x + 2y = 3$
$4x - y = -2$

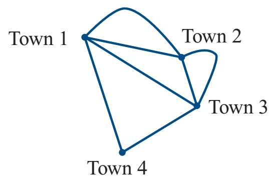
Extended-response questions

1 Each of the following diagrams represents the network of roads joining several towns. Represent each by a matrix A where the elements indicate the number of roads joining the towns.

a



b



c

2 Heights in feet and inches can be converted into centimetres using matrix multiplication. The matrix $C = \begin{bmatrix} 30.45 \\ 2.54 \end{bmatrix}$ can be used as a conversion matrix (one foot $=$ 30.45 cm and one inch equals 2.54 cm).

a What is the order of matrix C?

Jodie tells us her height is 5 feet 4 inches. We can write her height as a matrix $J = [5 \quad 4]$.

b What is the order of matrix J?

c Is the matrix product JC defined? Why?

d Evaluate the matrix product JC, and explain why it gives Jodie's height in centimetres.

e Matrix $H = \begin{bmatrix} 5 & 8 \\ 5 & 4 \\ 6 & 1 \\ 5 & 11 \end{bmatrix}$ gives the heights in feet and inches of four other people.

Use the conversion matrix C and matrix multiplication to generate a matrix that displays the heights of these four people in centimetres.

3

Number of titles	*Bookshop 1*		*Bookshop 2*	
	Hardback	*Paperback*	*Hardback*	*Paperback*
Fiction	334	876	354	987
Non-fiction	213	456	314	586

Books can be classified as *Fiction* or *Non-fiction* and come in either *Hardback* or *Paperback* form. The table shows the number of book titles carried by two bookshops in each of the categories.

a How many non-fiction paper back titles does Bookshop 1 carry?

b The matrix $A = \begin{bmatrix} 334 & 876 \\ 213 & 456 \end{bmatrix}$ displays the number of book titles available at Bookshop 1 in all categories. What is the order of this matrix?

(cont'd.)

Review

c Write down a matrix equivalent to matrix A that displays the number of book titles available at Bookshop 2. Call this matrix B.

d Construct a new matrix $C = A + B$. What does this matrix represent?

e The average cost of books is \$45 for a hardback title and \$18.50 for a paperback title. These values are summarised in the matrix $E = \begin{bmatrix} 45.00 \\ 18.50 \end{bmatrix}$.

  i What is the order of matrix E?

  ii Construct the matrix product AE and evaluate.

  iii What does the product AE represent?

f Bookshop 1 plans to move to larger premises that will enable it to double the number of titles it carries in every category. Write down a matrix expression that represents this situation and evaluate.