Section I

Question	Solution	Answer
1	A prism has a regular cross-section	D
2	$0 \le P(E) \le 1$	А
3	Time in Capetown = 1 pm $- 8$ hours $+ 2$ hours = 7 am	В
4	$ \tan 40^\circ = \frac{h}{15} $ $ h = 15 \tan 40^\circ $ = 12.586	С
5	$# ways = 3 \times 2 \times 1 = 6$	В
6	$y = a^x$ passes through (0,1) and is exponential	D
7	The scores have been stretched to the right	В
8	$-0.9 \Rightarrow$ downhill and linear	D
9	$x^{2} = 9^{2} + 12^{2}$ by Pythagoras' theorem = 225 $x = 15$	С
10	Days = $\underbrace{30 - 12 + 1}_{April} + \underbrace{20}_{May} = 39$ $I = 2100 \times \frac{19.74\%}{365} \times 39$ = 44.2933 Amount paid = $\$2100 + \44.29 = \$2144.29	С
11	Range $X = 9 - 3 = 6$; Range $Y = 11 - 5 = 6$. Each set of scores has the same range \Rightarrow B or D Mode $X = 9$; Mode $Y = 8$. The modes are different and X has the larger mode.	В
12	$\frac{6x^2y}{3} \div \frac{2y}{5} = \frac{6x^2y}{3} \times \frac{5}{2y}$ $= 5x^2$	А
13	$A = \frac{h}{3} (d_f + 4d_m + d_l) \text{ where } h = \frac{12}{4} = 3$ = $\frac{3}{3} (5 + 4(7) + 12) + \frac{3}{3} (12 + 4(8) + 10)$ = $5 + 4(7) + 2(12) + 4(8) + 10$ = 99	А

14	As there are 9 scores, there is a middle score and so the median is one of the scores. Two of the added scores are above the median and two are below and so the median will not change.	В
15	As the coin is unbiased, H and T are equally likely to occur. $\therefore P(\text{Tail in any toss}) = \frac{1}{2}$	В
16	3 years = $3 \times 26 = 78$ fortnights 9% pa = $\frac{0.09}{26}$ per fortnight	D
17	Original mean = $\frac{1.8 + 1.83 + 1.84 + 1.86 + 1.92}{5}$ = 1.85 \therefore New mean = 1.86 $\therefore 9.25 + x = 6 \times 1.86$ $x = 6 \times 1.86 - 9.25$ = 1.91	С
18	$s = ut + \frac{1}{2}at^{2}$ $\frac{1}{2}at^{2} = s - ut$ $at^{2} = 2(s - ut)$ $a = \frac{2(s - ut)}{t^{2}}$	А
19	Total pay = $40 \times \$22.35 + \$150 + (5+4) \times 2 \times \22.35 = $\$1446.30$	D
20	Difference = $$50000 - 20000 = $$30000$	С
21	Distance between A and $B = 2 \times 90$ km = 180 km Time for 2^{nd} train = 4 : 30 pm - 3 : 10 pm = 1 h 20 min \therefore Speed = $\frac{180}{1\frac{1}{3}}$ = 135	Α
22	MonthPrincipal (P)Interest (I)P + IP + I - R1\$250,000.00\$1,593.75\$251,593.75\$249,721.812\$249,721.81\$1,591.98\$251,313.79\$249,441.853\$249,441.85\$1,590.19\$251,032.04\$249,160.104\$249,160.10\$1,588.40	А

Section II Question 23

(a) Taxable income = $56\ 350 - 350 - 2\ 000 - 250$ = $53\ 750$

Medicare levy =
$$1.5\% \times $53750$$

= \$806.25

(ii) Pattern rule is
$$N = 5 + 3(F - 1)$$

or $N = 3F + 2$

:. when
$$F = 100$$
: $N = 3(100) + 2$
= 302

(iii) When
$$N = 543$$
: $543 = 3F + 2$
 $541 = 3F$
 $F = \frac{541}{3}$
 $= 180.\dot{3}$

It is not possible to draw the figure with exactly 543 sticks as the pattern number must be a whole number and it isn't.

(c) Value after 3 years =
$$1.340 \times $5000$$

= \$6700

(d) (i) Volume =
$$10000 \div 1000 \text{ m}^3 = 10 \text{ m}^3$$

(ii) Area of face
$$= \pi \left(\frac{1.34}{2}\right) \left(\frac{1.5}{2}\right)$$

 $= 0.5025\pi$
Now $V = Ah$
 $10 = 0.5025\pi h$
 $h = \frac{10}{0.5025\pi}$
 $= 6.3345...$

 \therefore the tank is approximately 6.33 m long to the nearest cm.

Question 24

- (a) (i) 900 mm
 - (ii) Bathroom = $2\ 000\ \text{mm} \times 2\ 000\ \text{mm}$
 - (iii) Length = $6485 + 3690 2 \times 240$ mm = 9695 mm
 - (iv) Length on the plan = 18 mm Scale of plan = 20 mm : 2 000 mm = 1 : 100 Width of window = 18 mm × 100 = 1 800 mm

(b) (i)
$$A = 72 - (16 + 11 + 8 + 12 + 15)$$

= 10

(ii) Relative frequency
$$=\frac{8}{72}$$

 $=\frac{1}{9}$

(iii) Expected number
$$=\frac{1}{6} \times 72$$

= 12

 \therefore 5 was rolled as many times as expected



 \therefore the bearing of *C* from *B* is 055°

=55°

(ii)
$$AC^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \times \cos 55^\circ$$

= 160.927...
 $AC = \sqrt{160.927...}$
= 12.685...
 $\doteqdot 13$ (nearest km)

(iii)
$$\frac{\sin\beta}{6} = \frac{\sin 114^{\circ}}{AC}$$
$$\sin\beta = \frac{6\sin 114^{\circ}}{12.685...}$$
$$= 0.43208...$$
$$\beta = 25.5997....^{\circ}$$
$$\doteq 26^{\circ} \text{ (nearest degree)}$$

: the bearing of A from C is $(180 + \theta + \beta)^\circ = (180 + 55 + 26)^\circ$ = 261°

Question 25

(a) (i) Categorical

(ii) Any question for which the answer is a number.e.g. How many text messages did you send today?

(iii) Choose the same number of students from each year group from all of the schools.

(iv) Survey every student in all NSW high schools.

(b) (i) Year 12 because 100% of those in the year group own a mobile phone.

(ii)
$$P(\text{Year 9 student has mobile}) = \frac{55}{70}$$

= 0.7857...
 $P(\text{Year 10 student has mobile}) = \frac{50}{60}$
= 0.83

 \therefore The Year 10 student is more likely to have a mobile phone.

(iii) Students in higher grades have proportionally more mobile phones.

(c) (i) Total =
$$330 + 250$$

= 580

(ii) P(Female chosen has pre-paid) = $\frac{172}{319}$ = 0.5391...

(iii) New percentage =
$$\frac{103+10}{261+10} \times 100\%$$

= 41.697...%
 \Rightarrow 42% (nearest whole percentage)

(d) (i) 71

(ii) There are 14 female students \therefore the median lies between the 7th and 8th scores.

There are 7 students below the median and 7 above the median.

: the lower quartile is the 4^{th} score = 11 and the upper quartile is the 4^{th} score from the highest = 20

 $\therefore IQR = 20 - 11$ = 9

Question 26

(a) (i) X = 2 + 3 = 5

(ii)
$$P(\text{score less than } 4) = \frac{6}{12}$$

 $= \frac{1}{2}$

(iii) To get a score of 3 if Spinner B already shows a 2, we need Spinner A to show a 1. P(a score of 3) = P(a 1 on Spinner A)

$$=\frac{2}{3}$$

(iv) Expected earnings = $\frac{4}{12} \times \$12 + \frac{1}{2} \times \$0 + \frac{2}{12} \times (-\$3)$ = \$3.50Cost to play = \$5Difference = \$5 - \$3.50= \$1.50

: she should expect a loss of \$1.50 per game played

(b) (i) Try t = 6

 $5 \times 3^6 = 3645$ Conclusion: t = 6 is too small

(ii) Try t = 8 $5 \times 3^8 = 32805$ Conclusion: t = 8 is too big Try t = 7 $5 \times 3^7 = 10935$ Conclusion: t = 7 is too small

 \therefore the population first exceeds 18 000 in the 8th year.

(c) Deposit =
$$15\% \times $20000$$

= \$3000

Balance owing =
$$$20000 - $3000$$

= \$17000

$$I = Prn$$

= 17 000 × 0.19 × 5
= 16150

Total to repay =
$$$17000 + $16150$$

= \$33150

Monthly repayment =
$$\frac{\$33150}{5 \times 12}$$
$$= \$552.50$$

Question 27



(i) I P $160^{\circ}W$ 0° $109^{\circ}E$

Difference in longitude = $160^{\circ} + 109^{\circ}$

= 269°

But this is the angle of the major arc as it is greater than 180°

$$\therefore \text{ angle of the minor arc} = 360^{\circ} - 269^{\circ}$$
$$= 91^{\circ}$$
Arc length = $\frac{91}{360} \times 2\pi \times 6400$
$$= 10164.79...$$
$$= 10165 \qquad (\text{nearest whole})$$

 \therefore the shortest distance is 10 165 km.

(ii)
$$\begin{array}{c} 48^{\circ} \\ \hline R & JI \\ 160^{\circ}W \\ \end{array} \begin{array}{c} P \\ 109^{\circ}E \end{array}$$

Longitude of Rabaul = $160^{\circ}W + 48^{\circ}W$ = $208^{\circ}W$ but this passes $180^{\circ}W$ by 28° Corrected longitude = $180^{\circ}E - 28^{\circ}$ = $152^{\circ}E$

 \therefore Rabaul is 4°S and 152°E

(c) (i) $z\text{-score} = \frac{400-500}{50}$ = -2

(b)

(ii) For Brand A, 400 has a *z*-score given by

$$z\text{-score} = \frac{400 - 450}{25}$$
$$= -2$$

As the *z*-score of 400 is -2 for both Brand A and Brand B, the proportion of bulbs lasting less than 400 hours will be equal in each of the distributions.

 \therefore the claim is false as the bulls are equally likely to be defective.

(d) J: \$50 000; 15 years; 6% pa

Josephine's financial gain = $$50\,000(1.06)^{15} - $50\,000$ = $$69\,827.90966...$ = $$69\,827.91$ (nearest cent) E: \$500 at the end of each month;

15 years = $15 \times 12 = 180$ months

6% pa = 0.5% per month

Emma's financial gain =
$$\$500 \left\{ \frac{(1.005)^{180} - 1}{0.005} \right\} - \$500 \times 180$$

= $\$55409.3562...$
= $\$55409.36$ (nearest cent)

 \therefore Josephine has the better financial gain after 15 years.

Question 28

(a) (i) $P = \frac{a}{V}$

(ii)
$$P = 3, V = 2$$

 $P = \frac{a}{V}$
 $3 = \frac{a}{2}$
 $a = 3 \times 2$
 $= 6$
 $\therefore P = \frac{6}{V}$

Now substitute V = 4: $P = \frac{6}{4} = 1.5$



- (ii) The gradient represents the value lost each year.
- (iii) S = 60000 4000n
- (iv) n < 0 is not suitable as this would refer to the time before the tractor was purchased.

n > 15 is not suitable as this is when the value would become negative, a situation which makes no sense in the problem.

(v)
$$S = V_0 (1-r)^n$$
 where $r = 0.2$; $V_0 = 60\,000$; $n = 14$
 $S = 60\,000 (1-0.2)^{14}$
 $= 2\,638.827...$
 \therefore the value was approximately \$2 639

(vi) As n > 15, the value continues to fall but at a slower and slower rate. It continues to drop towards zero but is always worth something, even if only very little.

End of solutions